Dynamics of jury competence with correlated judgments*

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Abstract

We examine how individual learning and group consensus interact in a dynamic extension of the Condorcet Jury Theorem by comparing the evolution of collective competence in juries with individual performance. We model individual learning using logistic functions and measure consensus through positive correlation between individual judgments. Short-term analysis compares the logistic and Condorcet trajectories near initial conditions, while long-term analysis examines how asymptotic solutions depend on model parameters.

Key Words: Condorcet Jury Theorem, correlated binary random variables, dynamic learning

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1 Introduction

The Condorcet Jury Theorem (CJT) provides a framework for collective decision-making under uncertainty. The theorem analyzes a group of competent jurors, where competence is defined as the probability of making correct decisions that exceeds random chance. Individual competence is the probability that a juror will make the correct decision alone. The collective competence of a jury is measured by the Condorcet probability (CP), or the probability that the jury collectively reach the correct verdict by simple majority. This binary framework applies to decision-making problems where actors must choose between two alternatives and possess judgment accuracy exceeding random chance.

The CJT states that the larger the group of competent and independent jurors, the greater the CP. The non-asymptotic version of the CJT asserts that the collective competence of an independent, homogeneous jury increases monotonically with the number of jurors and, in particular, that any jury with three or more independent and equally competent jurors is more competent than a single juror of equal competence. The asymptotic version of the CJT states that the jury becomes infallible (CP becomes a certainty) as the number of jurors tends to infinity.

The classic CJT requires jurors to be equally competent and independent. The equal competence assumption leads to a homogeneous jury, where individual votes are statistically identical (exchangeable), but may be correlated. The independence assumption becomes unrealistic when jurors share common sources of information or exhibit herding behavior due to similar backgrounds, leading to groupthink (Surowiecki 2005). Correlation undermines collective competence: perfectly correlated jurors are statistically equivalent to a single juror, which the classic CJT shows is inferior to an independent jury. Kaniovski and Zaigraev (2011) show that the non-asymptotic CJT does not hold for homogeneous juries with correlated votes. When juror pairs exhibit identical positive correlation coefficients, collective competence can fall below that of a single juror if individual competence is low while correlation is high. Enlarging an independent jury always improves its competence, whereas the same does not hold true for correlated juries, where competence may actually decline.

The standard CJT is static in that it treats individual competence measured by the likelihood of a correct individual decision as fixed during the process of reaching a collective verdict. There is no learning on the part of the jurors – their individual competence does not improve over time. In practice, both individual and collective competence tend to evolve over time through learning. Empirical evidence from behavioral sciences shows that learning curves often follow a logistic (sigmoidal) profile, by which initial performance improves first rapidly, then slows and eventually reaches a level set by the limit of cognitive capacity. A central parameter in a logistic learning model is the maximum achievable competence, or the highest probability that a juror makes the correct decision. This parameter captures the inherent difficulty of the problem and is assumed to be the same for all jurors. The assumption of fallibility maintains a positive error probability, ensuring that no juror becomes infallible by attaining perfect accuracy.

Using logistic learning profiles, we investigate whether the non-asymptotic CJT holds when competences evolve over time. We analyze a two-dimensional dynamical system that models the evolution of individual competence and consensus level, measured by the correlation between juror decisions. The dynamical system captures the effect of the interplay between the evolution of individual competence and the evolution of consensus on the collective competence measured by the CP. The solution of the dynamical system depends on the model parameters and the initial conditions given by the common initial competence of a given number of jurors and an initial correlation coefficient between them. We assume jurors start as independent (zero initial correlation) but become correlated as consensus emerges.

The proposed dynamic Condorcet jury setting integrates the temporal evolution of competence with the classic asymptotic analysis based on the size of the jury. This introduces temporal asymptotics in addition to the traditional asymptotics based on jury size. We compare the asymptotic (long-run) performance of the jury to that of a single logistic learner and then examine how this long-run solution varies with jury size. For the baseline parameterization of the model, we show that under fallibility (where individual competence remains below unity), jury collective competence may exceed individual performance only for juries of seven or more members. We establish that for juries of seven or more members, there is a critical jury size for which the asymptotic (long-run) collective competence of the jury exceeds that of a single juror. This implies that the traditional asymptotic Condorcet theorem, which relies on increasing jury size, holds for juries of seven or more members. The threshold for this critical jury size decreases with the inherent difficulty of the problem or as the maximum achievable competence decreases. However, this does not apply to smaller juries of three or five members, where the negative impact of positive correlation (groupthink) on collective competence leaves the single logistic learner asymptotically superior.

Our work is related to Mulligan (2024), who develops a variant of the CJT in which an opinion leader's influence simultaneously raises voter competence and creates dependence. Using the framework of Boland, Proschan and Tong (1989) with a single distinguished agent as the source of all dependence, Mulligan models competence and dependence as logistic functions of a static influence variable and shows that it is never optimal for the opinion leader to exert unbounded influence. Our model differs in several respects. First, we adopt the exchangeable Bahadur framework with symmetric jurors rather than the asymmetric opinion leader structure, reflecting settings in which consensus emerges endogenously among peers rather than being induced by a single agent. Second, our model features genuine temporal dynamics governed by an ODE system in which competence and consensus co-evolve over time, rather than being determined simultaneously as functions of a single choice variable. Third, the bidirectional feedback between competence and consensus – whereby high competence amplifies consensus formation and consensus in turn depresses competence – produces a self-reinforcing groupthink dynamic that drives the non-monotonic dynamics in our analysis. Finally, we address the asymptotic comparison between jury and individual performance as a function of jury size, establishing the existence of a critical jury size above which the jury is asymptotically superior to a single learner.

Although the classic static CJT originated in the social sciences (Young 1988, Boland 1989, Nitzan and Paroush 2019), it is relevant in many other contexts. The theorem and its extension to correlated inputs are applicable to many expert systems that involve binary decisions and correlated inputs, for example, in medical diagnostics (Srivastava, Pradhan and Saini 2022) or in the behavior of animal groups (Kao, Miller, Torney, Hartnett and Couzin 2014). Another example is binary classification in machine learning, where a positive correlation between learners affects the accuracy of the prediction and the learning speed of the model (Li, Paffenroth and Berthiaume 2021). Allowing for qualified majorities makes the CJT applicable to the k-out-of-n systems (Zuo and Tian 2010), where component redundancy interacts with system reliability, and positive correlation reflects failure cascades; see Zaigraev and Kaniovski (2010, 2013).

This paper is structured as follows. Section 2 develops the foundation by first reviewing the static homogeneous jury model with correlated votes, then extending it to a dynamic framework using logistic learning functions and a two-dimensional ordinary differential equation system that captures the joint evolution of individual competence and consensus over time. Section 3 examines the model's behavior through both short-term analysis near initial conditions and long-term asymptotic analysis, revealing conditions under which juries outperform individual decision-makers and identifying critical jury sizes for optimal collective performance. Section 4 offers concluding remarks.

2 The model

Our analysis builds on the CJT developed by Kaniovski (2009) and Kaniovski and Zaigraev (2011) for homogeneous juries with correlated votes. Their approach relies on a simplified version of the general parameterization introduced by Bahadur (1961) for the joint distribution of binary random variables. The simplification relies on two key assumptions: exchangeability, which ensures jury homogeneity, and truncation of the polynomial expansion of the probability to include only pairwise correlation coefficients (Pearson product-moment).[1] Although this truncation produces a parsimonious model amenable to analytical treatment, it requires constraining the correlation coefficient to ensure the existence of a valid distribution. We begin by describing this static framework before extending it to our dynamic analysis.

2.1 Static homogeneous jury

Let the number of jurors $n \geq 3$ be odd. A binary vector $\mathbf{v}$ of length n represents a voting outcome. In a homogeneous jury, the votes are exchangeable binary random variables. Exchangeability implies that the probability of $\mathbf{v}$ depends only on the total number of correct votes, $t_{\mathbf{v}} = \sum_{i=1}^{n} v_i$, rather than on the specific jurors who cast them. The homogeneous jury is therefore an example of a representative agent model, with all jurors being statistically equivalent. This reduces the n-dimensional problem to a single-dimensional one where only the total number of correct votes matters. Let p_i be the competence of the juror i, or the probability of her or his correct vote, and $c_{i,j}$ be the correlation coefficient between the votes of the juror i and the juror j, that is,

[1] For exchangeable binary random variables, more compact parameterizations of the joint probability distribution exist (e.g., George and Bowman 1995), though these do not explicitly characterize correlations.

p_i is the marginal probability of success and $c_{i,j}$ is the Pearson product-moment correlation coefficient. If $p_i = p \in (0.5, 1)$ for all $i = 1, \ldots, n$ and $c_{i,j} = c \in [0, 1)$ for all $1 \leq i < j \leq n$, then, under simple majority rule, the CP reads:

$$M_n(p,c) = I_p\left(\frac{n+1}{2}, \frac{n+1}{2}\right) + 0.5(n-1)c(0.5-p)\frac{\partial I_p\left(\frac{n+1}{2}, \frac{n+1}{2}\right)}{\partial p}, \tag{1}$$

where I_p denotes the regularized incomplete beta function. The above equation captures the collective competence of n equally competent but correlated jurors under simple majority rule.

The truncation constrains the correlation coefficient c for a given n and p:

$$c \leq \frac{2p(1-p)}{(n-1)p(1-p) + 0.25 - \mu}, \quad \text{where} \quad \mu = \min_{0 \leq t \leq n} \{[t - (n-1)p - 0.5]^2\} \leq 0.25. \tag{2}$$

The above constraint is bounded by the interval $(\frac{1}{n-1}, \frac{2}{n-1}]$. It is close to $\frac{1}{n-1}$ for large values of p, and close or equal to $\frac{2}{n-1}$ if p is close to 0.5. Since we are interested in positive correlations as a model of consensus, herding behavior, and groupthink, we assume $c \in [0, \frac{1}{n-1}]$.[2] This assumption is conservative, but avoids having to deal with an upper bound as a cumbersome function of p.

Two remarks on the truncation and the constraints are in order. First, we stress that the truncated Bahadur representation – in which all correlation coefficients of order three and higher are set to zero – yields a unique joint probability distribution for given marginal probabilities, pairwise correlations, and jury size n. The truncation is a modeling assumption, not a statement of ignorance about higher-order terms. In Appendix A, we derive the CP when third-order correlation coefficients are also retained, showing how the model can be extended beyond pairwise correlations.

Second, the tight upper bound on c is driven by exchangeability rather than by truncation per se. Exchangeability forces all $\binom{n}{2}$ pairwise correlations to equal the same value c, and a single parameter must simultaneously ensure non-negative probabilities for all 2^n outcomes (for a given p). This is highly constraining. When exchangeability is relaxed and more structure is imposed on the correlation matrix – for example, by specifying that dependence flows through a single agent as in the opinion leader model of Boland et al. (1989) and Mulligan (2024) – the constraints on admissible correlation values become less restrictive. To illustrate, consider $n = 3$ jurors with common competence p and suppose that jurors 1 and 2 are perfectly correlated ($c_{12} = 1$), so that they effectively act as a single agent. The remaining correlations $c_{13} = c_{23}$ are then constrained by $c_{13} \leq \frac{1}{2p}$ for $p \in (0.5, 1)$, which is less stringent than the upper bound implied by inequality (2) under exchangeability. For example, at $p = \frac{2}{3}$ the exchangeable bound from inequality (2) is $\frac{2}{3}$, whereas the structured bound is $\frac{3}{4}$. There is thus a tradeoff between the generality of the group model and the flexibility of the dependence parameters: exchangeability imposes minimal structure on the group but tightly constrains the correlation, while richer dependence structures relax the constraints at the cost of committing to a specific

[2] For a general version of inequality (2) for positive and negative correlations, see Kaniovski and Zaigraev (2011).

group structure. Our model deliberately adopts exchangeability to preserve the representative-agent framework and the analytical tractability it affords, including the closed-form equilibrium of the ODE system.

We now highlight key features of the above model. First, because the constraint (2) forces $c \to 0$ as $n \to \infty$, votes in a large homogeneous jury can only be weakly dependent. The model is therefore better suited to describe committees or expert groups rather than electorates, where dependence may be stronger and structured asymmetrically. Second, the simple model is inherently static, providing a cross-sectional perspective on how correlation affects collective competence. Third, the effect of positive correlation on collective competence is necessarily detrimental because for $p > 0.5$ the second term in equation (1) is negative. Finally and importantly, letting $n \to \infty$ in equation (1) subject to the constraint defined by inequality (2) renders the votes asymptotically uncorrelated and independent, whereby the model with correlation approaches the classic model with independent votes in which the CP is given by the upper tail of the binomial distribution. This is why we do not pursue an asymptotic version of the CJT with correlation as the jury size tends to infinity and instead focus on time asymptotics for a fixed jury size within a dynamic learning framework.

2.2 Dynamic learning

We model the evolution of individual competence over time using the logistic function. Logistic functions are widely used to model learning curves and knowledge diffusion. Logistic regression is also used to analyze the factors that influence knowledge acquisition or adoption. In the context of machine learning, for example, learning curves provide a graphical representation of how model performance improves with increasing training data or iterations. These curves often exhibit logistic growth, and the area under the curve reflects the accuracy of the model in active learning contexts (Viering and Loog 2023). Similarly, learning curves in social contexts demonstrate how human proficiency in a task develops as a function of the time invested in practice (e.g., a study of student learning by Koedinger, Carvalho, Liu and McLaughlin 2023).

The logistic function presents several advantages as a learning model. This S-shaped curve naturally captures the diminishing returns characteristic of learning, where initial rapid improvements gradually decelerate as learners approach their maximum potential. The properties of the logistic function ensure that the correctness probability increases monotonically and asymptotically approaches a maximum value, providing a realistic upper bound on achievable performance that reflects cognitive or contextual limitations. The growth implied by the logistic model aligns with empirical observations of accelerated learning during intermediate phases, while its built-in saturation mechanism prevents the unrealistic scenario of unbounded improvement. This combination of mathematical tractability, biological plausibility, and empirical validity makes the logistic function particularly appropriate for modeling competence dynamics.

In this paper, we adopt the following specification of a logistic learning function:

$$\widetilde{p}(t) = \frac{p_0 p_{\max}}{p_0 + (p_{\max} - p_0)e^{-\alpha p_{\max} t}}, \quad \alpha > 0,\ t \geq 0. \tag{3}$$

The logistic function $\widetilde{p}(t)$ represents the probability that a single juror will make the correct decision at the time t. Following the classic CJT framework, jurors are assumed to be competent – their individual probability of making the correct decision exceeds random guessing. We therefore constrain the initial competence $p(0) = p_0 \in (0.5, p_{\max})$, ensuring that the competence of a single juror begins above random guessing and can only improve over time.

Even with unlimited practice or exposure, individuals may remain imperfect decision-makers due to the complexity of the task or cognitive constraints. The parameter $p_{\max} \leq 1$ captures this idea of residual uncertainty, or the maximum attainable competence. The boundary case of $p_{\max} = 1$ corresponds to infallibility, which, while analytically convenient, is generally unrealistic for decision-making under uncertainty. The positive parameter α modulates the speed of learning: the higher α, the faster the convergence to maximum competence $p_{\max}$.

This individual logistic learning trajectory provides the foundation for analyzing the dynamics of collective competence. Under exchangeability, all jurors follow identical logistic trajectories, thereby preserving jury homogeneity. Under the additional assumption of independence, the identical trajectories allow direct comparison with the Condorcet trajectory that governs collective decision-making performance. In the full model with correlated competence, individual jury competence may exhibit non-monotonic behavior due to detrimental effects such as herding or groupthink, but the competence of independent jurors is assumed to improve monotonically according to the logistic specification above.

We now introduce a dynamic model that captures the evolving interaction between individual competence and group consensus, both of which influence the CP denoted by $M_n(p, c)$ and defined by equation (1). The model highlights that a higher level of group consensus – reflected in a positive correlation coefficient – has a detrimental effect on collective competence. Moreover, this negative effect is amplified when individual competence is high. In other words, the more competent the jurors, the greater the potential harm from herding behavior and groupthink.

The literature highlights several ways in which positive correlation as a statistical model of herding behavior or groupthink negatively affects collective competence by undermining individual learning. The positive correlation between agents undermines the advantage of aggregation, making the group act more like a single entity. Correlated judgments reduce collective wisdom and mask individual insights. This is the essence of groupthink discussed in Surowiecki's (2005) popular account and the reason why the literature on the CJT has sought to relax the independence assumption (e.g., Boland et al. 1989, Ladha 1992, Kaniovski 2009).

The results obtained in the context of CJT apply broadly, from political science to ensemble learning. For example, they closely mirror the issues with multicollinearity or correlated model errors in machine learning. In machine learning contexts, correlated models reduce the benefit of ensembling and increase systemic risk (e.g., Li et al. 2021, Li, Sarna and Lin 2025). Li et al.'s (2025) work emphasizes that error correlation is a pervasive and critical issue in multi-model ML deployments. Their framework provides practical tools to measure and manage these correlations, enabling more reliable risk estimates and better informed modeling decisions for ensemble and foundation model scenarios.

Similar results emerge in animal behavior. Kao et al. (2014) show that animal groups can outperform their best individuals through collective learning, aggregation, and updating of social information based on both personal and observed decisions. The authors show that the benefit of collective decisions breaks down when group members' information sources are too highly correlated. Vicente-Page, Pérez-Escudero and de Polavieja (2018) investigate how the size of the group affects the precision of collective decisions when individuals can dynamically review their choices. They find that larger groups, with more extensive information sharing, can inadvertently amplify shared errors, reinforcing the theme that positive correlation undermines collective wisdom.

Our formulation focuses on the self-enforcing effect of positive correlation (consensus, herding, groupthink) and individual competence while also allowing for consensus decay. Consensus and competence exhibit a self-reinforcing dynamic because highly competent agents demonstrate greater confidence in their judgments, which paradoxically increases their susceptibility to groupthink. When skilled individuals recognize their own reliability and observe similar competence in their peers, they interpret group agreement as confirmation of accuracy rather than as a potential indicator of correlated error. This mutual reinforcement creates a positive feedback loop that entrenches consensus by reducing incentives to challenge the prevailing opinions.

Empirical research in social psychology and decision science confirms that individual confidence increases conformity, particularly within expert groups where dissent faces social discouragement and appears unwarranted. Both theoretical models and real-world observations demonstrate that mechanisms designed to improve collective accuracy can instead foster homogeneity and suppress critical evaluation, thereby amplifying and perpetuating groupthink among highly competent agents.

Theoretical models, moreover, show that consensus, once established, becomes persistent due to social reinforcement and loss aversion: dissenters face pressure to rejoin the group, maintaining high correlation. However, consensus decay can occur when new information or disruptive events introduce sufficient heterogeneity or doubt – a dynamic common in deliberative bodies, markets, and animal collectives (Vicente-Page et al. 2018). New or surprising information can disrupt group alignment, breaking down consensus and reducing correlation.

To account for the self-enforcing effect between individual competence and consensus, we consider an ODE system that describes the joint evolution of juror competence $p(t)$ and consensus, measured by a positive correlation coefficient $c(t)$, over time $t \geq 0$:

$$\dot{p}(t) = \alpha p(t)[p_{\max} - p(t)] - \beta p(t)c(t), \tag{4}$$

$$\dot{c}(t) = \gamma p(t)\Big[\frac{1}{n-1} - c(t)\Big] - \delta c(t), \tag{5}$$

subject to the initial conditions: $p(0) = p_0 \in (0.5, p_{\max})$ and $c(0) = 0$. Here $p_{\max} \in (0.5, 1]$, while the dot denotes the time derivative.

Equation (4) describes the evolution of individual competence. It increases with experience and decreases as knowledge becomes obsolete. The term $\alpha p(t)[p_{\max} - p(t)]$ implies logistic learning, where competence grows with experience but saturates as it approaches $p_{\max}$. The parameter $\alpha > 0$ controls the rate of learning. The term $-\beta p(t)c(t)$ represents the negative impact of consensus on individual competence, suggesting that too much consensus (herding) can reduce individual competence by discouraging independent thinking and critical analysis. The parameter $\beta > 0$ modulates the strength of this effect.

Equation (5) describes the evolution of consensus. The consensus increases due to shared information or collaboration and decreases due to new evidence or independent reassessment. The term $\gamma p(t)\left[\frac{1}{n-1} - c(t)\right]$ captures the fact that the level of consensus increases with the competence of the jurors, but there is a limit $\frac{1}{n-1}$. The parameter $\gamma > 0$ represents the rate at which consensus accumulates in relation to competence. The term $-\delta c(t)$ implies that consensus diminishes over time, possibly due to changes in individual opinions or new contradictory evidence, with $\delta > 0$ controlling the rate of this decay. Note that plugging $c(t) = 0$ into (4) gives the logistic trajectory of a single juror given by equation (3). Without positive correlation, the model reduces to one of n independent and identically competent jurors, which can be directly compared to a single juror, similar to the approach of the classic CJT.

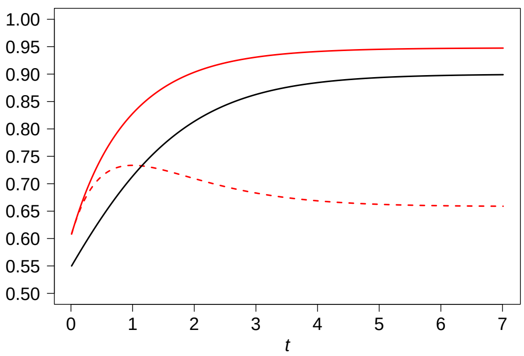
3 Analysis

The non-asymptotic version of the classic CJT compares the CP to the competence of a single juror. It asserts that a jury of competent and independent jurors will always outperform a single equally competent juror. In a dynamic context, we compare the Condorcet trajectory $M_n(p(t), c(t))$ with a logistic trajectory $\widetilde{p}(t)$ for $t \geq 0$. Logistic growth means that individual competence increases monotonically over time and converges asymptotically to $p_{\max}$. When $p_{\max} = 1$, a single juror with logistic learning eventually achieves perfect accuracy asymptotically. This is not the case for a correlated jury because in our model, correlation is detrimental to collective competence.

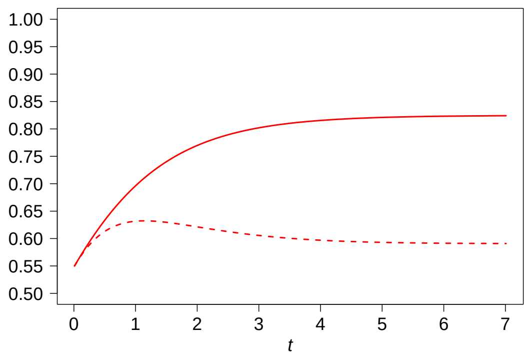
The dynamical system defined by equations (4) and (5) describes the joint evolution of individual competence $p(t)$ and consensus $c(t)$. This evolution depends on jury size n, five model parameters ($\alpha, \beta, \gamma, \delta \geq 0$ and $p_{\max} \in (0.5, 1]$), and two initial conditions ($p(0) = p_0 \in (0.5, p_{\max})$ and $c(0) = 0$). Figures 1-3 illustrate the model solutions for various parameter values and initial conditions. Figure 1 shows that both the Condorcet trajectory $M_n(p(t), c(t))$ and the individual competence trajectory $p(t)$ can be non-monotonic, initially increasing and then decreasing (red dashed line in panels (a) and (b)), if the detrimental effect of correlation is sufficiently strong (β is relatively large). Large values of β ensure that the logistic learning trajectory $\widetilde{p}(t)$ of the single juror overtakes the Condorcet trajectory in finite time, which does not occur when $\beta = 1$. Figure 2 shows that for sufficiently high initial individual competence, the individual competence trajectory $p(t)$ can be non-monotonic (red dashed line in panel (b)), yet the Condorcet trajectory remains monotonically decreasing (red dashed line in panel (a)). Figure 3 illustrates the effect of $p_{\max}$. Setting $p_{\max} = 1$ ensures that the single juror becomes asymptotically infallible and outperforms the jury (dotted lines in panel (a)). For moderate values of $p_{\max}$ ($p_{\max} = 0.8$), the jury retains its advantage over the single juror (dashed lines in panel (a)). In Section 3.2, we show that for the natural parameterization $\alpha = \beta = \gamma = \delta = 1$ this result does not hold for juries of sizes $n = 3$ and $n = 5$. For low values of $p_{\max}$ ($p_{\max} = 0.6$), the single juror again outperforms the jury at the final time point (solid lines in panel (a)).

The degree to which positive correlation is detrimental depends on the model parameters and initial conditions. Due to the complexity of the dynamical system, our analysis will focus on short-term and long-term perspectives rather than transitional dynamics. The short-term analysis describes the relationship between the logistic trajectory $\widetilde{p}(t)$ and the Condorcet trajectory $M_n(p(t), c(t))$ near the initial time $t = 0$, where $p(t)$ and $c(t)$ are close to p_0 and 0, respectively. The long-term perspective provides a sensitivity analysis of the asymptotic solution as $t \to \infty$.

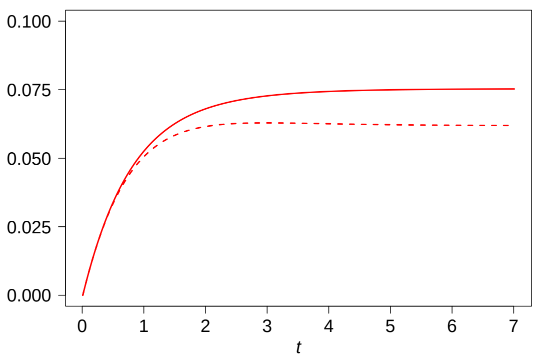
Figure 1: The effect of parameter β for $n = 7$.



(a) Condorcet $M_7(p(t), c(t))$ vs logistic $\widetilde{p}(t)$



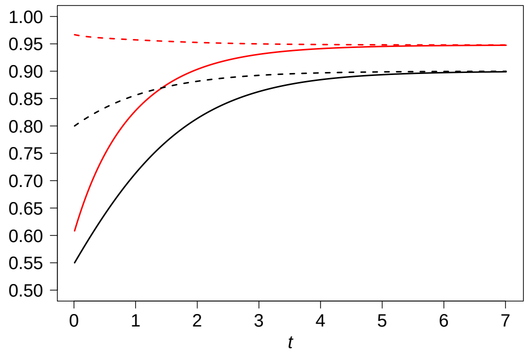
(b) Individual competence $p(t)$



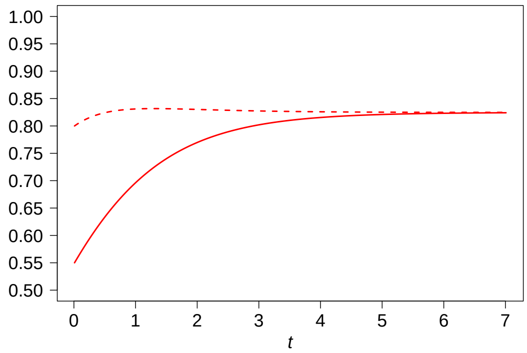
(c) Group consensus $c(t)$

Depending on the model parameters, Condorcet trajectories need not be monotonic in time t. The figure shows two Condorcet trajectories $M_7(p(t), c(t))$ (in red) for different parameter sets: $\alpha = \beta = \gamma = \delta = 1$ (solid line) and $\alpha = \gamma = \delta = 1$, $\beta = 5$ (dashed line). A higher value of β corresponds to a stronger negative effect of consensus on jury competence. Sufficiently strong detrimental effect of consensus can reduce the jury's performance over time, as measured by the Condorcet probability $M_7(p(t), c(t))$. Both trajectories assume $p_{\max} = 0.9$ and the initial condition $p_0 = 0.55$. For comparison, the logistic trajectory $\widetilde{p}(t)$, starting at $p_0 = 0.55$, is shown in black. This trajectory depends on α, but not on β. Panels (b) and (c) show the corresponding evolution of individual competence and group consensus.

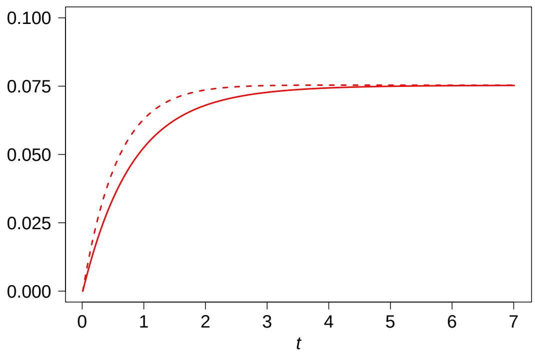
Figure 2: The effect of initial conditions for $n = 7$.



(a) Condorcet $M_7(p(t), c(t))$ vs logistic $\widetilde{p}(t)$



(b) Individual competence $p(t)$



(c) Group consensus $c(t)$

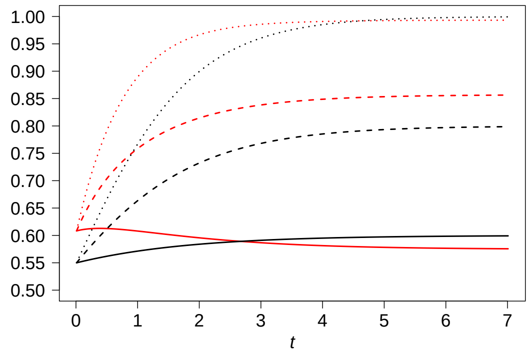
The monotonicity of Condorcet trajectories in time t also depends on the initial condition p_0. The figure shows two Condorcet trajectories $M_7(p(t), c(t))$ (in red) for different initial conditions: $p_0 = 0.55$ (solid line) and $p_0 = 0.8$ (dashed line). For sufficiently high initial competence, increasing consensus over time can reduce the jury's performance, as measured by the Condorcet probability $M_7(p(t), c(t))$. Both trajectories assume $\alpha = \beta = \gamma = \delta = 1$ and $p_{\max} = 0.9$. The corresponding logistic trajectories $\widetilde{p}(t)$, which depend on the initial condition but not on the level of consensus, are shown in black. Panels (b) and (c) show the corresponding evolution of individual competence and group consensus.

3.1 Short-term

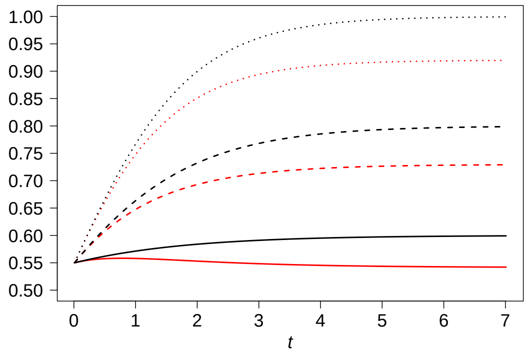
The short-term analysis focuses on the behavior of the CP near the initial conditions or when t is small. We begin by noting that $M_n(p_0, 0) = p_0$ when $p_0 = 0.5$. If the single juror and the jury members start with the individual competence $p_0 > 0.5$, then the jury starts with a higher collective competence, as $M_n(p_0, 0) > p_0$ for all $p_0 > 0.5$ (Kaniovski and Zaigraev 2011).[3] We examine whether the CP initially increases with time t, indicating that the jury members are

[3] For the truncated Bahadur distribution, $c = 0$ implies stochastic independence. This can be verified directly by substituting $c = 0$ in (1).

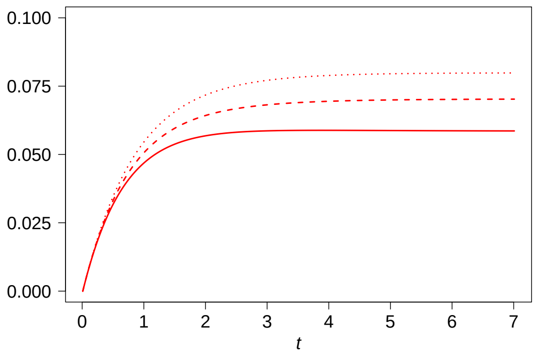
Figure 3: The effect of $p_{\max}$ for $n = 7$.



(a) Condorcet $M_7(p(t), c(t))$ vs logistic $\widetilde{p}(t)$



(b) Individual competence $p(t)$



(c) Group consensus $c(t)$

The figure shows three Condorcet trajectories $M_7(p(t), c(t))$ (in red) for different values of $p_{\max}$: solid line (0.6), dashed (0.8), and dotted (1). Each trajectory assumes $\alpha = \beta = \gamma = \delta = 1$ and the initial condition $p_0 = 0.55$. Setting $p_{\max} = 1$ or choosing $p_{\max}$ close to 0.5 (e.g., $p_{\max} = 0.6$) ensures that the logistic trajectory $\widetilde{p}(t)$ (in black) intersects the Condorcet trajectory (dotted and solid lines, respectively), meaning that a single juror eventually outperforms the jury as $t \to \infty$. For intermediate values (e.g., $p_{\max} = 0.8$), the Condorcet trajectory exceeds the logistic trajectory for $n = 7$, but not for smaller juries ($n = 3$ or $n = 5$). Under positive correlation, very small or highly incompetent juries will ultimately perform worse than a single juror with a logistic learning trajectory. Panels (b) and (c) show the corresponding evolution of individual competence and group consensus.

learning. Take the derivative of $M_n(p(t), c(t))$ as a function of t:

$$\dot{M}_n = \frac{(p(t) - p^2(t))^{\frac{n-3}{2}}}{B(\frac{n+1}{2}, \frac{n+1}{2})} \Big[(1 - 0.5c(t)(n-1))(p(t) - p^2(t))\dot{p}(t) \\ + 0.5(n-1)(0.5 - p(t))\dot{c}(t)(p(t) - p^2(t)) + 0.5c(t)(n-1)^2(0.5 - p(t))^2\dot{p}(t)\Big]. \tag{6}$$

Upon the substitution of $\dot{p}(t)$ and $\dot{c}(t)$ from (4) and (5) into (6), setting $t = 0$ we get

$$\dot{M}_n|_{t=0} = \frac{(p_0 - p_0^2)^{\frac{n-1}{2}} p_0}{B(\frac{n+1}{2}, \frac{n+1}{2})} \left[\alpha(p_{\max} - p_0) - 0.5\gamma(p_0 - 0.5)\right].$$

Therefore, $\dot{M}_n|_{t=0} > 0$ if and only if

$$p_0 < \frac{\alpha p_{\max} + 0.25\gamma}{\alpha + 0.5\gamma}. \tag{7}$$

For example, $p_0 < \frac{4p_{\max}+1}{6}$ for $\alpha = \gamma$. The right-hand side of (7) tends to $p_{\max}$ for $\frac{\alpha}{\gamma} \to \infty$, and to 0.5 for $\frac{\alpha}{\gamma} \to 0$. Thus, if the initial competence p_0 satisfies (7), then the jury members initially learn (the Condorcet trajectory increases from the very beginning), otherwise they unlearn. Interestingly, the behavior of $M_n(p(t), c(t))$ at the initial time (learn or unlearn) depends only on $p_0, p_{\max}$ and $\frac{\alpha}{\gamma}$, but not, for example, on n.

The above analysis can be generalized to qualified majority as a decision-making rule on the dynamics of collective competence (Appendix B). A qualified majority changes the CP by introducing a quota $q = \frac{k-1}{n-1}$. This quota represents the number of correct votes k required to reach the correct collective decision. The case $k = \frac{n+1}{2}$ corresponds to the simple majority.

3.2 Long-term

Let $a = \frac{\beta}{\alpha}$ and $b = \frac{\delta}{\gamma}$. The system (4)-(5) has three stationary points (p^*, c^*). The first is the origin, whereas the other two points are given by:

$$c^* = \frac{\frac{a}{n-1} + b + p_{\max} \pm \sqrt{D}}{2a}, \text{ where } D = \Big(\frac{a}{n-1} + b + p_{\max}\Big)^2 - \frac{4ap_{\max}}{n-1} > 0, \text{ and } p^* = p_{\max} - ac^*. \tag{8}$$

Both the origin and the equilibrium point corresponding to $+\sqrt{D}$ lie outside the relevant region $(0, p_{\max}) \times [0, \frac{1}{n-1}]$. This leaves the point corresponding to $-\sqrt{D}$ as the unique equilibrium of the model. It is locally asymptotically stable for all positive parameter values and attracts any trajectory that originates in $p(0) = p_0 \in (0.5, p_{\max})$ and $c(0) = 0$. In the limit, the competence of a jury member depends on the size of the jury $n, p_{\max}$ and the parameters a and b, but not on the initial competence p_0:

$$p^* = p_{\max} - ac^* = \frac{p_{\max} + \sqrt{(\frac{a}{n-1} + b + p_{\max})^2 - \frac{4ap_{\max}}{n-1}} - [\frac{a}{n-1} + b]}{2} \in (0, p_{\max}), \tag{9}$$

while

$$c^* = \frac{p_{\max} - \sqrt{(\frac{a}{n-1} + b + p_{\max})^2 - \frac{4ap_{\max}}{n-1}} + [\frac{a}{n-1} + b]}{2a} \in \left(0, \frac{1}{n-1}\right).$$

It follows that p^* increases monotonically in $n, p_{\max}$ and b and decreases monotonically in a. Correlation can severely impair both individual and collective competence, with p^* potentially falling below 0.5 or even approaching 0. The limit value for the level of consensus c^* increases monotonically in $p_{\max}$ and decreases monotonically in n, a and b. Finally, $p^* \to p_{\max}$, $c^* \to 0$, as $n \to \infty$.

The equilibrium competence p^* depends on $a = \beta/\alpha$ and $b = \delta/\gamma$, which govern the detrimental effect of consensus on individual learning and the rate of consensus decay, respectively. These parameters enter the equilibrium asymmetrically: a appears as $a' = \frac{a}{n-1}$, so its effect is modulated by jury size, whereas b enters independently of n. Large a combined with small b corresponds to a setting where consensus strongly harms individual competence and decays slowly (a "groupthink trap"), while small a combined with large b corresponds to a setting where consensus decays quickly and the system behaves nearly like independent learners with p^* close to $p_{\max}$.

We now examine the range of p^* more precisely.

1. *When does* $p^* < 0.5$*?* The equilibrium competence falls below random guessing if and only if
$$p_{\max} < \frac{1}{2} + \frac{a'}{1+2b}. \tag{10}$$
This condition is always satisfied when $a' - b > \frac{1}{2}$, regardless of $p_{\max}$. Otherwise, whether $p^* < 0.5$ depends on the value of $p_{\max}$. For $a = b = 1$, condition (10) becomes $p_{\max} < \frac{1}{2} + \frac{1}{3(n-1)}$, which is satisfied only for $p_{\max}$ very close to 0.5 when n is large.

2. *Can* p^* *approach 0?* We have $p^* \to 0$ if and only if $b \to 0$ with $a' \geq p_{\max}$, or $a' \to \infty$. In either case, consensus decay vanishes relative to its harmful effect on competence. This does not arise under the baseline parameterization $a = b = 1$.

3. *What is the infimum of* p^* *under the baseline parameterization?* For $\alpha = \beta = \gamma = \delta = 1$, we have $\inf p^* = \frac{\sqrt{3}-1}{2} \approx 0.366$, achieved in the limiting case $p_{\max} = 0.5$ and $n = 3$.

Table 1 illustrates how p^* varies with a, b, and n for $p_{\max} = 0.8$.

Table 1: Equilibrium competence p^* for $p_{\max} = 0.8$ and selected values of a, b, and n.

a	b	$n=3$	$n=5$	$n=7$	$n=9$
1	10	0.764	0.782	0.788	0.791
1	1	0.610	0.697	0.730	0.747
10	1	0.150	0.269	0.359	0.426

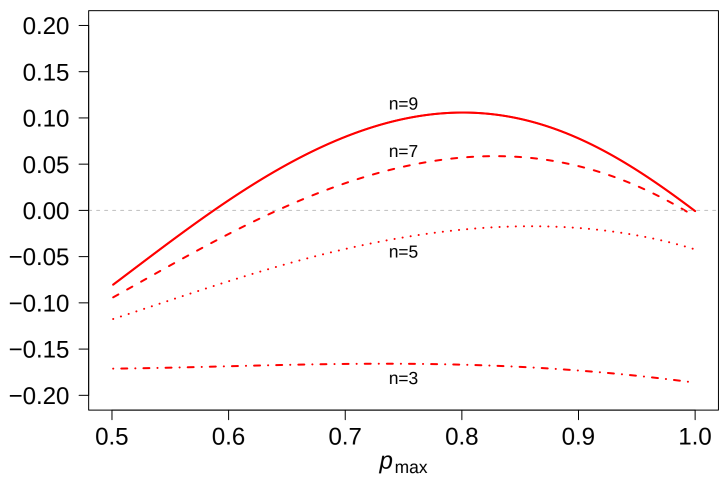
The table shows p^* as a function of a, b, and jury size n. When consensus decays quickly relative to its harmful effect on competence ($a = 1$, $b = 10$), the equilibrium competence remains close to $p_{\max} = 0.8$. When consensus is both harmful and persistent ($a = 10$, $b = 1$), equilibrium competence drops sharply; for $n = 3$ it falls to 0.150, well below random guessing. Larger juries mitigate the effect because the upper bound on c decreases with n.

The long-term analysis follows directly from the equilibrium (8). The logistic trajectory $\widetilde{p}(t) \to p_{\max}$ monotonically as $t \to \infty$ and will eventually exceed the Condorcet trajectory for $p_{\max} = 1$, because $M_n(p(t), c(t)) \to M_n(p^*, c^*) < 1$. So, the two trajectories always intersect. If $p_{\max} < 1$, the two trajectories may also intersect, although they are not required to do so. If they intersect, let T be the intersection time. For the main case, where $M_n(p(t), c(t))$ increases monotonically in t, we can compute the upper bound of the intersection time by solving $\widetilde{p}(t) = M_n(p^*, c^*)$ and obtaining the solution
$$T^* = \frac{1}{\alpha p_{\max}} \ln \left[\frac{(p_{\max} - p_0) M_n(p^*, c^*)}{p_0 (p_{\max} - M_n(p^*, c^*))} \right].$$

The competence of an individual juror definitely exceeds the collective competence of the jury after T^*, but it can also exceed it at an earlier point.

Let us compare $M_n(p^*, c^*)$ to $p_{\max}$ for the case $p_{\max} \in (0.5, 1)$ – or when the single juror does not become asymptotically infallible – for the parameterization $\alpha = \beta = \gamma = \delta = 1$. Figure 4 plots the difference between the asymptotic probability $M_n(p^*, c^*)$ and $p_{\max}$ for several values of n. The figure shows that $M_n(p^*, c^*) < p_{\max}$ for $n \leq 5$ (Appendix C for a proof). For $n \geq 7$, the interval $(0.5, 1)$ is split into two parts: for $p_{\max} \in (p_1, p_2)$, we have $M_n(p^*, c^*) > p_{\max}$, while for $p_{\max} \in (0.5, p_1) \cup (p_2, 1)$, we have $M_n(p^*, c^*) < p_{\max}$. Here, p_1 monotonically decreases in n to 0.5 as $n \to \infty$, while p_2 monotonically increases in n to 1 as $n \to \infty$.

Figure 4: The asymptotic comparison of a Condorcet jury and a single juror.



The figure shows the difference between $M_n(p^*, c^*)$ and $p_{\max}$ for different jury sizes. The competence of a single juror, given by the logistic trajectory $\widetilde{p}(t)$, converges to $p_{\max}$ as $t \to \infty$. Under the baseline parameterization $\alpha = \beta = \gamma = \delta = 1$, a jury with fewer than seven members is always asymptotically inferior to a single juror following a logistic learning trajectory. With at least seven jurors, the jury becomes asymptotically superior, unless $p_{\max}$ is close to 0.5 or close to 1.

To summarize, given $p_{\max} \in (0.5, 1)$ there exists a critical jury size n_{cr}, such that for all $n \geqslant n_{cr}$ the jury is asymptotically superior to a single juror, or $M_n(p^*, c^*) > p_{\max}$. This existence result holds for any parameter values. To see this, note that $M_n(p^*, c^*) = M_n(p_{\max} - ac^*, c^*)$, where $c^* = c^*(n, p_{\max})$. Since $p^* \to p_{\max}$ and $c^* \to 0$, as $n \to \infty$, for any $p_{\max} \in (0.5, 1)$, we have $M_n(p^*, c^*) \to 1$ as $n \to \infty$, for any $p_{\max} \in (0.5, 1)$. In the case $\alpha = \beta = \gamma = \delta = 1$ we can make the existence result more precise by showing that $n_{cr} \geqslant 7$ (Appendix C).

4 Conclusions

We develop a dynamic version of the Condorcet Jury Theorem that incorporates the interaction between individual learning and group consensus, comparing jury collective competence with individual performance. The emergence of consensus over time induces a positive correlation between individual juror votes. Excessive consensus (groupthink, herding behavior) impairs group performance relative to an individual logistic learner, with correlation-induced harm increasing as juror competence rises.

We first compare the Condorcet and logistic trajectories near the initial conditions. Under simple majority rule with $p_0 > 0.5$, an initially uncorrelated jury always outperforms a single juror, consistent with the classic non-asymptotic result. The Condorcet trajectory starts above the logistic trajectory. While the logistic trajectory grows monotonically, near the initial conditions both individual juror competences and the collective Condorcet trajectory may increase or decrease depending on model parameters. We can summarize these findings as follows:

1. If $p_0 > 0.5$, the Condorcet trajectory starts above the logistic trajectory.
2. For small values of p_0, the growth rate of the Condorcet trajectory at $t = 0$ exceeds that of the logistic trajectory. As p_0 increases, the difference between the initial growth rates of the two trajectories decreases. Eventually, the growth rate of the Condorcet trajectory becomes smaller than the logistic growth rate, and may even become negative, indicating deteriorating performance, for values of p_0 greater than $\frac{\alpha p_{\max}+0.25\gamma}{\alpha+0.5\gamma}$.
3. If $p_{\max} = 1$, which corresponds to the case of an asymptotically infallible single juror, the two trajectories always intersect. In this case, the single juror will eventually outperform the correlated jury. In the more realistic case $p_{\max} < 1$, the intersection might not exist. This happens, e.g., when the Condorcet trajectory exceeds the logistic trajectory asymptotically.

The asymptotic analysis shows that if the maximum individual competence is 1 (the infallibility assumption), then a positive correlation makes the jury asymptotically inferior to a single juror. If the maximum individual competence is less than unity, the jury can be asymptotically inferior or superior to a single juror. In the baseline parameterization, the difference between jury and individual competence remains negative when the maximum competence approaches either 0.5 or 1. Under this parameterization for any maximum competence below unity, a single juror remains asymptotically superior to small juries (sizes 3 and 5), but may be asymptotically inferior to juries of size not less than 7, confirming the previous result for static correlated juries that a substantial enlargement of the jury is needed for collective competence to benefit. The sensitivity analysis of the equilibrium competence reveals that large $a = \beta/\alpha$ combined with small $b = \delta/\gamma$ can push equilibrium competence well below random guessing, particularly for small juries, while small a combined with large b keeps the system close to the performance of independent learners. The effect of a is modulated by jury size, whereas the effect of b is not.

A Third-order Condorcet probability

In this appendix, we derive the CP when the truncated Bahadur representation retains both second-order and third-order correlation coefficients. Let c denote the common pairwise (second-order) correlation coefficient and r the common third-order correlation coefficient under exchangeability. Then the CP generalizes to:

$$M_n(p,c,r) = M_n(p,c,0) + \binom{n-1}{\frac{n-1}{2}} (p(1-p))^{\frac{n-2}{2}}\, r \binom{n}{3} \left[\frac{n-3}{4(n-2)} - p(1-p)\right].$$

Equivalently, in terms of the regularized incomplete beta function:

$$\begin{aligned} M_n(p,c,r) =& I_p\left(\frac{n+1}{2}, \frac{n+1}{2}\right) \\ &+ \frac{\partial I_p\left(\frac{n+1}{2}, \frac{n+1}{2}\right)}{\partial p} \left(\frac{c(n-1)}{2}\left[\frac{1}{2} - p\right] + \frac{r(n-1)(n-2)}{6(p(1-p))^{\frac{1}{2}}}\left[\frac{n-3}{4(n-2)} - p(1-p)\right]\right). \end{aligned}$$

The effect of r on the CP depends on the sign of the term $\frac{n-3}{4(n-2)} - p(1-p)$. Since $p(1-p)$ is maximized at $p = \frac{1}{2}$ where it equals $\frac{1}{4}$, and $\frac{n-3}{4(n-2)} < \frac{1}{4}$ for all finite n, the term is negative when p is close to $\frac{1}{2}$ and positive when p is sufficiently far from $\frac{1}{2}$ (i.e., close to 1 in the competent-juror regime). For $n = 3$, the term equals $-p(1-p) < 0$ for all $p \in (0.5, 1)$, so that positive third-order correlation is always detrimental to the CP in a three-member jury.

B Qualified majority

In this appendix, we consider the effect of qualified majority as a decision-making rule on the dynamics of collective competence. The qualified majority rule introduces a quota $q = \frac{k-1}{n-1}$ representing the number of correct votes k required to reach the correct collective decision, where the case $k = \frac{n+1}{2}$ corresponds to the simple majority.

Under qualified majority, the CP (1) generalizes to:

$$P_n^k(p,c) = I_p(k, n-k+1) + 0.5(n-1)c(q-p)\frac{\partial I_p(k, n-k+1)}{\partial p}.$$

If $q \in (0.5, 1)$, then $P_n^k(p_0, 0) < p_0$ for $p_0 \in (0.5, \bar{p})$ and $P_n^k(p_0, 0) > p_0$ for $p_0 \in (\bar{p}, 1)$. The point $\bar{p}$ is the fixed point for the cumulative distribution function of $B(k, n-k+1)$ distribution. It depends on q and increases from 0.5 to 1 with k. In other words, given $q \in (0.5, 1)$ there exists a threshold $\bar{p}$ such that if the single juror and the jury members begin with $p_0 > \bar{p}$, then the two trajectories will intersect, if $p_{\max} = 1$. However, intersections may also occur in other cases.

Furthermore, $\dot{P}_n^k|_{t=0} > 0$ if and only if

$$p_0 < \frac{\alpha p_{\max} + 0.5\gamma q}{\alpha + 0.5\gamma}.$$

This inequality generalizes inequality (7). To summarize,

1. For $p_0 \in (0.5, \bar{p})$, the Condorcet trajectory (jury competence) starts below the logistic trajectory (single juror competence), whereas for $p_0 \in (\bar{p}, 1)$ it starts above. The threshold $\bar{p}$ increases to 1 as q increases to 1.

2. The growth rates of the two trajectories depend essentially on the model parameters. The growth rate of the Condorcet trajectory at $t = 0$ is positive for values of p_0 smaller than $\frac{\alpha p_{\max} + 0.5\gamma q}{\alpha + 0.5\gamma}$ and negative (unlearn) for values of p_0 larger than this level. If q increases, then in most cases the difference between the initial growth rates of the Condorcet and the logistic trajectories first increases and then decreases.

3. The two trajectories do not necessarily intersect.

Under unanimity, when $q = 1$, the Condorcet trajectory starts below the logistic trajectory. The initial growth rate of the Condorcet trajectory is positive, but no intersection point exists, if $p_{\max} = 1$.

C Cases $n \leq 5$

We begin by establishing several useful formulas. From equation (9), it follows that

$$p_{\max} = \frac{p^*(p^* + \frac{a}{n-1} + b)}{p^* + b},$$

and, therefore,

$$c^* = \frac{1}{n-1} \cdot \frac{p^*}{p^* + b}. \tag{11}$$

Let $\alpha = \beta = \gamma = \delta = 1$. Then,

$$p^* = p^*(p_{\max}, n) = \frac{p_{\max} - \frac{n}{n-1} + \sqrt{(p_{\max} + \frac{n}{n-1})^2 - \frac{4p_{\max}}{n-1}}}{2},$$

and

$$p^*(1, n) = \frac{\sqrt{n^2 - 2n + 1.25} - 0.5}{n-1}, \quad p^*(0.5, n) = \frac{\sqrt{9n^2 - 14n + 9} - (n+1)}{4(n-1)}. \tag{12}$$

Case $n = 3$.

For any $p_{\max} \in (0.5, 1)$, we would like to show that

$$M_3(p^*, c^*) = 3p^{*2} - 2p^{*3} - 3c^* p^*(1 - p^*)(2p^* - 1) < p_{\max}.$$

Since $p_{\max} = p^* + c^*$, we can rewrite the last inequality as

$$p^*(1 - p^*)(2p^* - 1)(1 - 3c^*) < c^*.$$

We now show that for any $p_{\max} \in (0.5, 1)$,

$$c^* - p^*(1 - p^*)(2p^* - 1)(1 - 3c^*) > 0. \tag{13}$$

Substituting c^* from equation (11) with $b = 1$ into (13), we obtain the inequality to prove:

$$\frac{0.5p^*}{p^* + 1} - p^*(1 - p^*)(2p^* - 1)\frac{1 - 0.5p^*}{p^* + 1} > 0. \tag{14}$$

We need to establish inequality (14) for $p^* \in (p^*(0.5, 3), p^*(1, 3))$, where from equations (12), $p^*(0.5, 3) \approx 0.366$ and $p^*(1, 3) \approx 0.781$.

The left-hand side of inequality (14) can be written as

$$\frac{p^* f(p^*)}{p^* + 1},$$

where $f(x) = -x^3 + 3.5x^2 - 3.5x + 1.5$. Inequality (14) follows from the positivity of the function

f in the interval $(p^*(0.5, 3), p^*(1, 3))$, as

$$\min_{x \in (p^*(0.5,3), p^*(1,3))} f(x) \approx 0.42.$$

Case $n = 5$.

For any $p_{\max} \in (0.5, 1)$, we have

$$M_5(p^*, c^*) = 10p^{*3} - 15p^{*4} + 6p^{*5} - 30c^*(2p^* - 1)p^{*2}(1 - p^*)^2 < p_{\max},$$

which is equivalent to

$$p^*(1 - p^*)(2p^* - 1)[1 + 3p^*(1 - p^*)(1 - 10c^*)] < c^*.$$

The proof of this inequality follows the same approach as for the case $n = 3$. We show that for any $p_{\max} \in (0.5, 1)$,

$$c^* - p^*(1 - p^*)(2p^* - 1)[1 + 3p^*(1 - p^*)(1 - 10c^*)] > 0. \tag{15}$$

Substituting c^* from equation (11) with $b = 1$ into (15), we obtain the inequality to prove:

$$\frac{0.25p^*}{p^* + 1} - p^*(1 - p^*)(2p^* - 1)\left[1 + 3p^*(1 - p^*)\frac{1 - 1.5p^*}{p^* + 1}\right] > 0. \tag{16}$$

We need to prove inequality (16) for $p^* \in (p^*(0.5, 5), p^*(1, 5))$, where from equations (12), $p^*(0.5, 5) \approx 0.425$ and $p^*(1, 5) \approx 0.883$.

The left-hand side of inequality (16) can be written as

$$\frac{p^* g(p^*)}{p^* + 1},$$

where $g(x) = 9x^5 - 28.5x^4 + 35x^3 - 17.5x^2 + x + 1.25$. The minimum of the function g in $(p^*(0.5, 5), p^*(1, 5))$ is positive

$$\min_{x \in (p^*(0.5,5), p^*(1,5))} g(x) \approx 0.04,$$

which establishes inequality (16).

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