Community Size, Heterogeneity and Voter Turnouts*

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Abstract

Numerous studies have found a negative relationship between the closeness of an election, the size of the electorate and voter turnout. It is often claimed that this relationship supports the rational voter hypothesis, with closeness and size proxying for the decisiveness of a vote. We offer a different interpretation. Larger communities are more heterogeneous than smaller ones, and turnouts are inversely related to the heterogeneity of a community. We present empirical support for this hypothesis using data for voter turnouts in Norwegian school language referendums. Community size is found to have a negative effect on voter turnouts, even after accounting for the probability of a single vote being decisive and the linguistic dimension of this heterogeneity. Our findings question the adequacy of the turnout regression in testing the rational voter hypothesis, as neither a positive correlation between closeness and turnout, nor a negative correlation between size and turnout can be exclusively attributed to instrumental voting.

JEL-Codes: D70, D72, C16
Key Words: voter turnout, community size, heterogeneity

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1 Introduction

Ever since Anthony Downs (1957) discussed the behavior of a rational voter in the context of a two-party system, the Downsian rational voter hypothesis has been a thorn in the side of the empirically minded public choice scholar. If the rational voter votes instrumentally, as virtually all of the public choice literature assumes, then the probability of his voting should depend on the difference between the expected benefits and costs of voting. The expected benefits in turn equal the benefit to the voter of having his preferred alternative, candidate or party win, times the probability that his vote is decisive in bringing about this outcome. As this probability is virtually zero in electorates of even moderate size, the rational voter hypothesis is routinely rejected by turnout rates in excess of 70 percent in national elections around the world. Hence the "paradox of voting" arises when one assumes that voters are rational and vote instrumentally.

A second paradox arises in studies that attempt to empirically test the rational voter hypothesis. A vote is decisive when it creates or breaks an exact tie. Owen and Grofman (1984) show the probability of a single vote being decisive in a two-way election to be a rather complicated nonlinear function of the size of the electorate and the fraction of voters who will vote for candidate 1. The smallness of this explicit measure of decisiveness presents an obvious empirical difficulty. The common practice of separating the probabilistic effect of closeness from that of size may seem a way to circumvent this difficulty. However, we argue that it is an unsatisfactory one as far as testing the rational voter hypothesis is concerned. When closeness and size are entered *separately* into turnout regressions, they indeed often return statistically significant coefficients of the predicted signs, the evidence on closeness being more consistent than that on size.[1] Voters' decisions to participate in elections thus seem to accord with the rational voter

[1] Surveys of empirical tests of the rational voter hypothesis are available in Matsusaka and Palda (1993), Blais (2000, ch. 1) and Mueller (2003, ch. 14.2).

hypothesis, *if one assumes that they greatly overestimate the probabilities of their votes being decisive*. This empirical success is remarkable, although the turnout regression only delivers the effect of a change in closeness and size on voter turnout. Explanatory variables alone cannot account for the absolute level of the dependent variable, which is one reason to include a constant term.[2]

Downs' (1957) suggestion for resolving the voting paradox stipulated that people vote because of a sense of civic duty. In this case, voters may be gratified by the act of voting rather than the outcome.[3] In other explanations for why people vote, voting is seen as an ethical act, or it is simply assumed that citizens have a desire to express their preferences in an election.[4] These various hypotheses offer plausible ways around the first paradox, but at the same time they underscore the second paradox. Why should the utility an individual gets from expressing her preferences or undertaking an ethical act vary with the size of the electorate or the closeness of the election?[5]

This article proposes a resolution to the second paradox by offering a rationale for the empirical success of the turnout regression, a rationale that rests on its ability to capture factors unrelated to the efficacy of the vote. We show that the closeness and size of the electorate remain significant in a turnout regression even after one has controlled for the efficacy of a vote by including the proper measure of efficacy - the probability that a vote is decisive. The

[2]See Grofman (1993) on the "marginalist defense" of the rational voter hypothesis and its discussion in Blais (2000, pp. 10-11) and Dowding (2005).

[3]For a classic article on this theme, see Riker and Ordeshook (1968).

[4]The ethical voter hypothesis was first put forward by Goodin and Roberts (1975). Fiorina (1976) first proposed the expressive voter hypothesis, although it has received its most extended development by Brennan and Lomasky (1997), Brennan and Hamlin (2000) and Schuessler (2000). A discussion of the importance of ethical motivations can be found in Blais (2000), who also provides favorable survey evidence.

[5]Existing models of non-instrumental voting do not suggest a systematic relationship between closeness, size and turnout. For example, Schuessler (2000, ch. 6) shows that imputing voters with both instrumental and expressive motivations leads to a non-monotonic relationship between size and turnout, as large electorates confer potentially large expressive benefits but strip the vote of all power. Edlin, Gelman and Kaplan (2005) show the expected utility of voting to be independent of the size of the electorate when voters have social preferences. While being proportional to the size of the community, the utility derived from imposing the desired election outcome on others is balanced by the probability that the vote is decisive.

main hypothesis of this article is that both closeness and electoral size reflect *heterogeneity* in the electorate, and that heterogeneity has a negative influence on electoral participation. This explains why these variables remain significant even in regressions using data from large electorates, where the probability of a vote being decisive is close to zero. As our test of this hypothesis rests on our being able to control for efficacy using the probability of a vote being decisive, we need to run the test using sufficiently small electoral districts, so that this probability is significantly above zero for most electorates. Our data source satisfies this criterion.

Matsusaka and Palda (1999) and Kirchgässner and Schulz (2005) offer an alternative explanation for why closeness is significant in turnout regressions. They argue that closeness measures the intensity of electoral competition, and thus reflects the pressure put on citizens to vote rather than the perceived efficacy of their vote. Close elections would then lead to greater voter mobilization, which would cause high electoral turnouts. Empirical studies show that closeness loses explanatory power in turnout regressions once mobilization efforts, often measured by campaign spending, are controlled for.[6]

Although we recognize the importance of voter mobilization in national elections, and in some referendums in Switzerland and California, we do not believe that it has played an important role in the Norwegian language referendums that constitute our data base. To the best of our knowledge, there has been no large scale campaigning in association with these referendums. Sometimes public meetings were held prior to referendums, and people wrote letters to the local press advocating their point of view, but no organized interest groups appear to have been involved in trying to turn out the vote.

The remainder of the article is organized as follows. In the next section we present the data on

[6]Rosenstone and Hansen (1993) offer a comprehensive study of voter mobilization in the United States. Further empirical evidence is available from the Swiss referendums in Kirchgässner and Schulz (2005). For a further discussion, see Aldrich (1995), Aldrich (1997) and Schachar and Nalebuff (1999).

voter turnouts in Norwegian school language referendums. In Section 3 we review the formulas for the probability of a single vote deciding a two-way election and estimate a turnout regression using the Norwegian data that includes this probability as the key explanatory variable. In Section 4 we review the literature on how the heterogeneity of a community may affect the willingness of people to participate in various activities, and present the rationale behind two alternative measures of community heterogeneity. The first is a Herfindahl-Hirschman index. Such measures have been used in many studies, and here we have constructed the index to specifically measure the degree of *language* heterogeneity in a school district. We also show that the Herfindahl-Hirschman index of heterogeneity is highly correlated with three measures of closeness commonly employed in turnout studies.

Our second measure of heterogeneity is community size, which we show to be able to capture other dimensions of heterogeneity not explicitly measured with a Herfindahl-Hirschman index. In Section 5 we revisit the turnout regression and estimate the elasticity of turnout with respect to the Herfindahl-Hirschman measure of linguistic heterogeneity, the size of the electorate and the probability of a vote being decisive. Briefly anticipating our results, we find that all three variables are significant and have the predicted effects on participation. These results provide support for both the rational voter hypothesis and the importance of heterogeneity for voter turnouts. The last section offers some concluding observations.

2 The Data

The key to our effort to disentangle the effects of closeness and electorate size as measures of heterogeneity is to include an exact measure of the probability of a vote being decisive in a turnout equation. As this probability is infinitesimal in large electorates, it is essential that we turn our attention to small electorates, where it is realistic to expect instrumental motivations.

The majority of empirical studies derive specifications based on the probability that one vote will decide an election offering only two alternatives. These two considerations point to local referendums as the best source of data for testing the rational voter hypothesis.

Since data on small electorates are rare, our choice of samples to test the model was limited. We used data from 232 Norwegian school district referendums on the choice of language for each district carried out between 1971 and 2003. The sizes of the electorates in these referendums were suitably small, ranging from 6 to 4,625 with an average of 395 eligible voters (see Table 1 and Figure 1). In addition to the small electorates and the binary choice, there were at least two other reasons that made the Norwegian data an attractive choice for our purpose. First, as a medium of direct democracy, a referendum is less prone to influences related to political representation. In a school language referendum we may, therefore, expect less confusion on the voter's part as to what is at stake than we would expect in elections for representatives of a legislature. The existence of a relationship between closeness, size and turnout becomes a moot issue once we admit the possibility that rational people might vote for reasons other than instrumental ones. Admitting the presence of several voter motivations begs the question of which conditions promote which type of behavior. We assume that referendums in small communities where the voters face an unambiguous binary choice are ideally suited for determining whether voters vote instrumentally, while large, mass media assisted national elections are more likely to foster voter participation based on other motives - as, for example, those claimed under the expressive and ethical voter hypotheses, and due to voter mobilization by political parties and special interest groups. Second, there is arguably less racial, ethnic and religious heterogeneity in Norway than in the USA, which has been the focus of most prior studies. Comprehensive data on the exact degree of heterogeneity are, to the best of our knowledge, unavailable for the time period covered.

Even if such data were available they would only describe heterogeneity from the vantage point of the characteristics that were actually sampled.

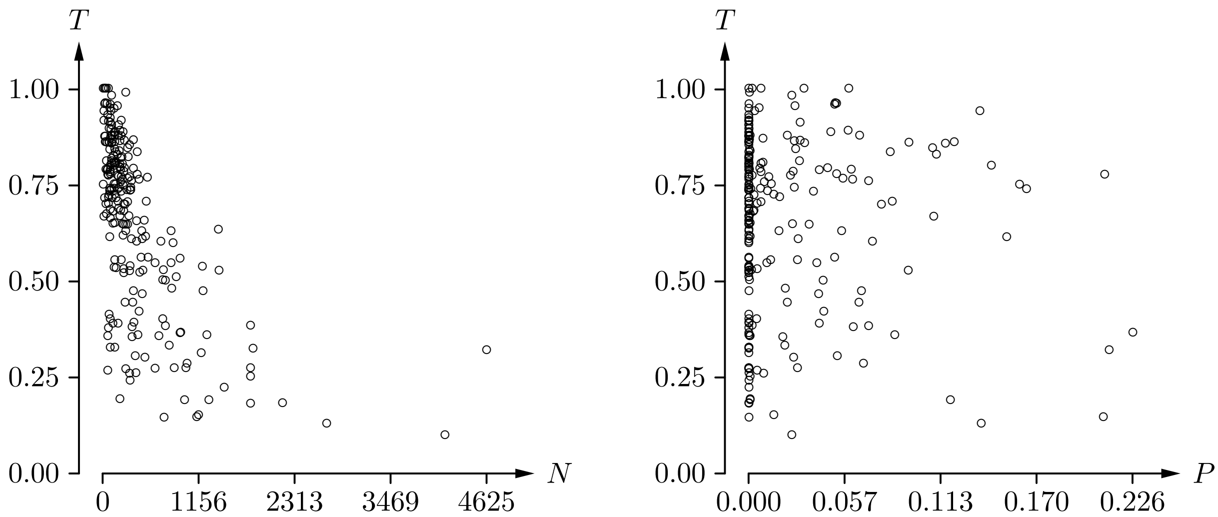
Table 1: Descriptive statistics

		Obs.	Mean	St.Dev.	Min.	Max.
		OUTCOMES OF THE REFRENDUMS				
Turnout	T	232	0.66	0.23	0.10	1.00
Electorate	N	232	394.98	561.48	6	4625
Split	s	232	0.47	0.18	0	0.89
Probability of decisive ballot	P	232	0.02	0.03	0	0.16
		LINGUISTIC HETEROGENEITY IN SCHOOLS				
Share Nynorsk-using pupils	n	192	0.23	0.20	0.0002	0.59
Share Bokmål-using pupils	$1-n$	192	0.77	0.20	0.41	1.00
Herfindahl-Hirschman Index	$HH(n)$	192	0.28	0.17	0.0004	0.50
		MEASURES OF CLOSENESS				
Quadratic	$Q(s)$	232	0.03	0.05	0	0.25
Absolute Value	$A(s)$	232	0.14	0.12	0	0.50
Entropy	$En(s)$	229	0.63	0.11	0.05	0.69

In each school district the electorate chose which of Norway's two official languages, Bokmål or Nynorsk, should constitute the primary language of the school district. Prior to 1985 the referendums had a semi-binding character, where the prefix "semi" refers to the condition that at least one of the alternatives had to be supported by 40 percent of the electorate in order for the outcome to be binding. Since 1985 all referendums, of which there were 86 in total, have only been advisory. All outcomes of advisory referendums have been implemented by the Norwegian authorities. In some referendums only the parents of children in the schools were allowed to vote. Søberg and Tangerås (2004) show, both analytically and using the data described above, that turnout is higher in a semi-binding referendum, because a vote in a semi-binding referendum is more decisive than in an advisory referendum. The parents of children in schools appear to be more concerned than the general public about the language their children are taught, as extending the franchise to parents only increased voter turnout in these referendums. Of more relevance to our study is the finding that both the size of the electorate in the school district and the expected closeness of the outcome proved to be good predictors of voter turnout.

The average turnout of 66 percent is typical of referendums held separately from legislative elections in other countries reported in Blais (2000, Table 1.8). With an average votes split of 47 percent, defined as the share of votes in favor of Nynorsk in total votes, the referendums were also on average quite close. This is important because the formulas for the probability of a single ballot deciding the outcome of the referendum are approximations for splits close to one half.

Figure 1: Turnout (T), electoral size (N) and the probability of a ballot being decisive (P)



The coefficient of correlation between turnout and the size of the electorate is -0.60; between turnout and the probability of a single ballot deciding the referendum it is 0.36. Both coefficients differ from zero at the 1 percent level of significance.

3 Testing the Downsian Rational Voter Hypothesis in Small Electorates

The rational voter hypothesis as first formulated by Anthony Downs might be stated as follows

$$T = B \cdot P - C \ , \tag{1}$$

where T is the probability that a citizen votes, B is the benefit that she gets from having her preferred outcome win, P is the probability that her vote is decisive in bringing about the victory of her preferred outcome, and C is the cost of her voting.[7] In a national election, the perceived differences between two candidates or parties may differ greatly across citizens, and thus tests of the Downsian voter model have appropriately tried to measure differences in B using answers to questions about the differences between candidates. In our empirical test of this model, each voter faces a binary choice over which of Norway's two official languages, Nynorsk or Bokmål, should constitute the primary language of her school district. It seems reasonable to assume that the benefit that a citizen gets from having her preferred language win will be about the same, regardless of whether she prefers Nynorsk or Bokmål. Thus, in the particular context in which we test the rational voter hypothesis, it seems reasonable to assume that the B factor drops out, since it is roughly the same for all voters. Since the electoral districts are all quite small, the cost of voting is also unlikely to vary greatly across the districts leaving P, the probability that a vote is decisive, as the key explanatory variable. There is one category of voters, however, for whom the benefits from having the referendum go their way may be greater than for other citizens. Parents are likely to get more benefits from having their children educated in their

[7] See Downs (1957, chs. 11-14) and, for further developments, Tullock (1967, pp. 110-114) and Riker and Ordeshook (1968).

mother tongue than citizens without children. Thus, when the voting franchise is restricted to parents, we expect increased turnouts. In this particular case we thus allow for differences in B.

Let p be the probability that a ballot is cast in favor of Nynorsk. Under the binomial assumption on the distribution of the voting poll, if the size of the electorate N is odd, then a single ballot would decide the outcome of the referendum approximately with probability

$$P(Decisive\ Ballot) = \frac{2\exp(-2(N-1)(p-0.5)^2)}{\sqrt{2\pi(N-1)}} \ . \tag{2}$$

For an even N this probability depends on how ties are resolved. If ties are broken by a fair coin flip, then

$$P(Decisive\ Ballot) = \frac{2\exp(-2N(p-0.5)^2)}{\sqrt{2\pi N}} \ . \tag{3}$$

Formulas (2) and (3) are Stirling's approximations of the exact probability, further simplified for p close to one half. The term $(p-0.5)^2$ is a measure of closeness. For a given N the probability that a single ballot decides the referendum is the highest when $p = 0.5$ and falls rapidly as p diverges from one half.

In empirical work p is commonly approximated by the actual split of votes. We also follow this practice, but note that it implicitly assumes the absence of any systematic biases in voters' anticipations about the closeness of a referendum. The split is the simplest frequency-based and objective measure of closeness. It is objective in the sense that it is a population statistic. The term "subjective measure of closeness" is typically reserved for survey evidence on how voters perceive closeness *ex ante*. The actual split is, of course, an *ex post* measure.

The Downsian model suggests a linear relationship between turnout and the probability of a vote being decisive. As a robustness check we also estimate a log-linear specification, which can be viewed as an approximation to more general nonlinear functional forms, and is a useful baseline model as such. Taking the logarithm has the additional benefit of reducing the skewness in the distribution of the dependent variable apparent from the left panel of Figure 1.

As a further robustness check, we estimate both specifications using conditional median regression. The objective function of a conditional median regression minimizes the sum of absolute values of the residuals instead of the sum of squares of the residuals, as in the case of OLS. The estimates of conditional median regression are thus more robust to outliers. Inference methods available to conditional quantile regression, of which conditional median regression is a special case, are also more robust to distributional assumptions on the error process.[8]

Table 2: Turnout as a function of the probability of casting a decisive vote

Dependent Variable	Turnout		Dependent Variable	log(Turnout)	
Regression Type	OLS[1]	Median[2]	Regression Type	OLS[1]	Median[2]
SEMI-BINDING = 1	0.23	0.26	SEMI-BINDING = 1	0.35	0.39
	(8.04)	(6.81)		(6.90)	(6.85)
	***	***		***	***
PARENTS = 1	0.18	0.23	PARENTS = 1	0.34	0.41
	(4.00)	(2.94)		(4.97)	(4.17)
	***	***		***	***
P(DECISIVE BALLOT)	2.01	1.48	LOG(P(DECISIVE BALLOT))	0.01	0.01
	(6.31)	(3.68)		(5.80)	(3.72)
	***	***		***	***
CONSTANT	0.47	0.50	CONSTANT	-0.55	-0.55
	(21.34)	(13.26)		(-10.97)	(-9.77)
	***	***		***	***
Obs.	232	232	Obs.	217	217
R^2	0.33	0.24	R^2	0.41	0.26

[1]Robust Standard Errors; [2]Bootstrap Standard Errors, Pseudo R^2;
Numbers in parenthesis are t-statistics; *** 1 percent; ** 5 percent; * 10 percent level of significance.

Table 2 presents four estimates of the turnout regression with P as an explanatory variable.

[8]Conditional quantile regression, of which conditional median regression is a special case, was first proposed by Koenker and Bassett (1978) and has been constantly improved since. This method has found applications in consumer theory, finance, and environmental studies, and is becoming an increasingly popular alternative to the OLS estimation of conditional mean models. Koenker (2005) provides a comprehensive survey of recent developments of this technique.

The first two enter T and P linearly and present standard OLS and as a robustness check conditional median regressions. The second two take the logs of both variables. We have also included two dummy variables. First indicating the type of referendum, whether advisory or semi-binding and, second, whether only the parents of children in the schools could vote. In all four regressions, the coefficient on the P variable is highly significant. When estimated by OLS, the linear version of the model returns the coefficient on P which is greater than one ($F(1, 228) = 10.09$). This suggests that some nonlinear form of the model is to be preferred, since the turnout in an election cannot exceed 100 percent. The coefficient on P in the conditional median regression is, however, not significantly different from one ($F(1, 228) = 1.41$). Despite this difficulty of the coefficient on P being greater than one, the results in Table 2 suggest that the decision to vote in Norwegian language referendums was strongly affected by the probability of a citizen's vote being decisive. Coefficients on the dummy variables pick up the effect of increased decisiveness in semi-binding referendums, and the differences in Downsian B between the parents and the rest of the electorate. The two effects are similar in terms of their quantitative impact on turnout. We now turn to a discussion of the importance of the heterogeneity of the community to voter turnout.

4 Heterogeneity and Participation

A. Background

Numerous empirical studies have looked at the impact of personal characteristics and the embedding social environment on the individual's participation in civic and social affairs. Evaluations of U.S. community surveys show that better educated and wealthier citizens pay more attention to local and national politics and are more likely to show up at the polls (Aldrich (1997)).

These individuals are also the ones who are more likely to meet at local town and school meetings (Campbell (2005)), or to be members of voluntary social clubs (Alesina and La Ferrara (2000)). The same evidence also reveals that people strongly prefer to interact with their like in terms of cultural and economic background. The average level of civic engagement in local communities falls with increasing ethnic, racial, and income heterogeneity. Individuals are more reluctant to provide or pay for public goods like education or welfare when the recipients' ethnicity or race differs from their own or the recipients are poor (Poterba (1997); Alesina, Baqir and Easterly (1999); Luttmer (2001)). Although most of these studies focus on the USA, the observed phenomena are by no means unique to that country. A handful of studies has also revealed the detrimental effect of heterogeneity in local communities in other Western democracies (Lindert (1996)) and in developing countries (La Ferrara (2002); Miguel and Gugerty (2005, forthcoming)), and the importance of heterogeneity seems likely to become one of the stylized facts of the participation literature as more data for other countries become available. Indeed, the observed phenomena are by no means unique to our epoch, as the study of desertion from the Union Army during the American Civil War by Costa and Kahn (2003b) suggests. The bottom line drawn in the recent survey of this literature by Costa and Kahn (2003a) is that cultural and economic heterogeneity reduces the willingness of the individual to associate with and trust other members of her community. Similar conclusions have been reached by political scientists, for example, in Rallings, Temple and Trasher (1994). By undermining social trust and eroding what may be termed social capital, heterogeneity makes it more difficult to find solutions that serve all community members equitably and efficiently. Crime, economic underdevelopment and a lack of supply of public goods seem to be symptomatic of communities with little social capital. Thus, the extent of an individual's participation can be seen to depend on both the kind of person she is and the type of community to which she belongs. The effects of community on an individual's behavior are known as *contextual* influences. In contrast with an individual's personal characteristics, such as race, education, religion or income, contextual factors capture the influence of the social environment.

Logically, it seems reasonable to expect heterogeneity to also have a detrimental effect on electoral participation. Nevertheless, there are several reasons not to expect this conclusion to be universally true. While commonality is characteristic of the membership in voluntary social associations, political competition usually embodies conflict and as such may excite participation. In a comprehensive study of U.S. local politics, Oliver (2001) has shown this to often be the case, although Rubenson (2004), for example, finds racial heterogeneity to influence electoral participation differently for different racial groups. Oberholzer-Gee and Waldfogel (2005) argue that citizens are more likely to participate in elections if they belong to large and presumably more heterogeneous groups, because economies of scale are involved in using media to mobilize them.

However, from a purely instrumental point of view one might also argue that citizen participation in the political process will decline with increased homogeneity as a result of the vote losing all instrumental power. Differences of opinion with respect to an issue or a candidate on the ballot should lead to closer outcomes, thus increasing the efficacy of a vote. Furthermore, participation might decline with increasing homogeneity because of the falling external costs of collective action.[9] High societal consensus effectively reduces the risk of an undesired collective outcome being imposed on an individual, thus making freeriding by abstention attractive to her. In examining arguments for and against low electoral participation in homogeneous societies, it seems that the existence and direction of an effect depends on the context and is a largely empirical issue.

[9]See Buchanan and Tullock (1962, pp. 115-116) and, in particular, Kafoglis and Cebula (1981).

B. Measuring Heterogeneity Using a Herfindahl-Hirschman Index

The empirical literature on non-electoral participation has focused on the impact of individuals' characteristics and the contextual effect of heterogeneity. Most of these empirical studies have chosen particular sources of heterogeneity - race, religion, or ethnicity - and then divided the community into different groups using these categories. The heterogeneity of the community is then measured using a Herfindahl-Hirschman index of inequality. We follow this practice by computing a Herfindahl-Hirschman index of linguistic heterogeneity. Starting from 1974 we have data on the number of pupils whose main language is Nynorsk or Bokmål, and 192 observations in total (Table 1). We use these data to compute the Herfindahl-Hirschman index of linguistic heterogeneity, as

$$HH(n) = 1 - n^2 - (1-n)^2 \ , \tag{4}$$

where n denotes the share of pupils whose main language is Nynorsk. The index takes values on the closed interval $[0, 0.5]$, with the value of zero corresponding to the maximum homogeneity and the value of one half corresponding to maximum heterogeneity. Unfortunately these data are only available on the county level, and may therefore be prone to aggregation biases when used on the school district level. These are the only available data on heterogeneity with direct pertinence to the issue on the ballot, however.

C. Closeness and Heterogeneity

As we noted in the introduction, measures of the expected closeness of an election have frequently been used to test the rational voter hypothesis, the notion being that the probability of a person's vote being decisive is higher, the closer the election is expected to be. But in large electorates, this probability is close to zero, *even in close elections.* Nevertheless, the closeness variable is often significant, even for large electorates. We contend that this may arise because there is a strong relationship between the usual measures of closeness and the heterogeneity of a community, which we now demonstrate.

In the formulas for the probability of a vote being decisive given in equations (2) and (3), the closeness of the electoral contest appears as the quadratic $Q(s) = (s - 0.5)^2$. Two alternative measures of closeness are frequently used in empirical literature: the absolute value $A(s) = |s - 0.5|$ and the entropy measure $En(s) = -s\log(s) - (1-s)\log(1-s)$ proposed by Kirchgässner and Schimmelpfennig (1992). The absolute value measure is equivalent to the margin of victory statistic, which has, for example, been used in Matsusaka and Palda (1993). The margin of victory is defined as the absolute difference in the number of votes in favor of each alternative over total votes cast. Compared to the quadratic measure, the absolute value puts more moderate weight on s's that are far from one half. The entropy measure is a positive and convex function of s. It attains a unique maximum at one half, around which the function is symmetric. This measure differs from the other two in terms of the sign of its effect on turnout, which is positive. Note that, unlike the other measures, it is not defined for $s = 0$ or $s = 1$ when the expected outcome is unanimous. Our sample includes three such cases, hence the lower observation count for this variable in Table 1. One advantage of the entropy measure is that it can easily be generalized to three or more alternatives.

If voters vote sincerely and are representative of their community, then the split of votes, s, in a referendum should accurately represent the fraction of community that prefers Nynorsk as the main language of their school district. This makes a Herfindahl-Hirschman index, which is computed using the actual split of votes, a valid measure of linguistic heterogeneity in the school district. We now formulate

Proposition 1 *Let s be the votes split in an election with two alternatives and $HH(s)$ the Herfindahl-Hirschman index. $Q(s)$, $A(s)$, and $En(s)$ are, respectively, the quadratic, the absolute value and the entropy measures of closeness. Then,*

(a) $HH(s)$ and $Q(s)$ are collinear with the coefficient of correlation $\rho(HH, Q) = -1$,

(b) $HH(s)$ and $A(s)$ are nearly collinear with $\rho(HH, A) \approx -0.936$ under the binomial assumption on the distribution of the voting poll with $p = 0.5$, and

(c) $HH(s)$ and $En(s)$ are nearly collinear with $\rho(HH, En) \approx 1$ for $p \approx 0.5$.

While a complete proof of Proposition 1 is tedious and has therefore been included in the Appendix, we may pause for a few remarks on these results. Parts (a) and (c) are obtained by algebraic manipulation. Part (a) is distribution free and holds for any pair of random variables for which the coefficient of correlation is defined.[10] Parts (b) and (c) are asymptotic results for $N \to \infty$, or as $s \to p$. Part (b) yields the coefficient of correlation computed under the binomial assumption appropriate to our setting. The underlying analytical result is, however, essentially distribution free and allows us to compute the coefficient of correlation for any distribution with mean 0.5 and finite variance.

Our data clearly support the hypothesis of perfect or near perfect collinearity between the three measures of closeness and the Herfindahl-Hirschman index of heterogeneity as $\rho(HH, Q) = -1$, $\rho(HH, A) \approx -0.946$, $\rho(HH, En) \approx 0.997$, all being insignificantly different from plus or minus unity at the 95 percent level.

Taking the natural logarithm of the probability of casting a decisive vote leads to a log-linear specification which is frequently estimated in the literature. In this specification closeness and size appear separately. A perfect collinearity between the quadratic measure of closeness and

[10] That is, for any pair of random variables with non-degenerate distributions and finite second moments.

the Herfindahl-Hirschman index of heterogeneity means that plugging one instead of the other into a turnout regression will produce a set of estimates identical up to the constant and the coefficient on the substituted variable. By looking at the estimates of such turnout regressions, we will not be able to determine whether they are produced by the expected closeness or the heterogeneity with respect to the issue on the ballot.

The Herfindahl-Hirschman index measures a particular dimension of heterogeneity - income, race, religion or, as in our study language. Since the issue to be decided in the referendums is one of language, we have arguably measured the most salient dimension of heterogeneity with respect to these referendums. Most of the variables that have been used to measure heterogeneity, such as income, are not available at the school district level at which the referendums occur. Moreover, there are a myriad other dimensions of heterogeneity on which people may differ or disagree. Thus, it would be desirable to have one variable which captures these additional, unmeasured dimensions of heterogeneity. In the next subsection we show that the size of a community, or more precisely the logarithm of size, can serve as such a variable.

D. Community Size and Heterogeneity

Consider a community of N individuals, each possessing a characteristic $u_i \in \mathbb{R}$, $i = 1, \ldots, N$ in which heterogeneity plays a role as a contextual variable. In our empirical application u_i may be the utility an individual i receives from her children having Nynorsk as their main language at school, with $u_i = 0$ signifying her indifference.

Given the vector of individual characteristics, $\{u_N\} = u_1, \ldots, u_N$, we seek to express the expected distance between the extreme characteristics $\mathsf{E}(\max\{u_N\} - \min\{u_N\})$ as a function of the community size N. To aggregate individual characteristics we make three distributional assumptions. First, we assume that $u_1, \ldots, u_N$ are *independent* random variables with probability distribution functions $F_1, \ldots, F_N$, so that the minimum is distributed as[11]

$$\Phi(x) = 1 - \prod_{i=1}^{N} \left[1 - F_i(x)\right] . \tag{5}$$

Second, that a scaling parameter $a_i > 0$ and a shift parameter b_i exist such that

$$F_N(x) = F(a_i(x - b_i)) , \tag{6}$$

or that the probability distribution functions F_i are identical up to a linear transformation of the argument. *Scaling* may seem a strong distributional assumption, but is less so when we are primarily interested in the actual realizations rather than the underlying distributions; the former can greatly vary given sufficient variance in the latter. Together, *independence* and *scaling* imply

$$\Phi(x) = 1 - \prod_{i=1}^{N} \left[1 - F(a_i(x - b_i))\right] . \tag{7}$$

Third, to express the expected distance between the extreme characteristics as a function of the community size N we assume that F is *minimum stable* with respect to the class of distributions defined by the *scaling* assumption. Then, by definition $\Phi(x) = F(a_N(x - b_N))$. Gumbel (1958) showed that the actual distribution for the minimum or the maximum realization in a random sample drawn from the same distribution converges to one of the three limiting distributions, if the initial distribution is minimum-stable. For $u_i \in \mathbb{R}$ the functional form of F must be of Type I (or Gumbel) extreme value distribution, $F(x) = 1 - \exp(-\exp(x))$.[12] This

[11] Stochastic independence can be replaced by weak stochastic dependence known as the Strong Mixing Property - see Leadbetter, Lindgren and Rootzén (1983, ch. 14.2). This weaker notion would then state that preferences become almost independent as we move apart in the spectrum of all preferences. For example, in the spectrum of political views, preferences of left and right would then be essentially independent, but those of more moderate members would correlate.

[12] For details see Leadbetter et al. (1983, ch. 14.2) and Arnold, Balakrishnan and Nagaraja (1992, ch. 8.3).

key distributional result defines F as a function of N. To see this, substitute into the Type I functional form so that $\exp(a_N(x - b_N)) = \exp(a_1(x - b_1 + \log(N)/a_1))$, where $a_N = a_1$ and $b_N = b_1 - \log(N)/a_1$. By setting $a_1 = 1$ and $b_1 = 0$ we obtain

$$F(x) = 1 - \exp(-\exp(x + \log(N))) \ , \tag{8}$$

from which the expected value for the minimum is obtained by integration

$$\mathsf{E}\min\{u_N\} = -(\gamma + \log(N)) \ , \tag{9}$$

where γ is Euler's constant, $\gamma \approx 0.58$. Repeating the same argument for the maximum yields

Proposition 2 *In a community of N individuals each possessing a characteristic $u_i \in \mathbb{R}$, $i = 1, 2, \ldots, N$ the expected distance between the extreme characteristics increases as the natural logarithm of N*

$$\mathsf{E}(\max\{u_N\} - \min\{u_N\}) = 2(\gamma + \log(N)) \ . \tag{10}$$

It follows that heterogeneity, in the sense of the distance between the extremes, increases less than proportionally with size or, in other words, the larger and presumably more diverse the community, the smaller the effect admitting a new member on this diversity.

Numerous studies have found community size to have a negative effect on non-electoral participation (Table 3). Smaller communities and smaller jurisdictions seem to be conducive to higher participation rates. Proposition 2 provides a theoretical justification for this finding. Heterogeneity increases with community size, and heterogeneity has a negative impact on participation. Of course, community size may be, in the words of Rose (2002), a "surrogate measure of other characteristics". The most obvious candidate from the public choice literature is the probability that a voter's vote is decisive. We eliminate this possible role for community size by including an explicit measure of the probability that a single vote is decisive in the model to be tested. Whatever community size captures in our regressions, it is not the probability of a vote being decisive, as this is captured directly by the inclusion of an estimate of this probability in the equation. We claim that community size captures otherwise unmeasured dimensions of community heterogeneity. We now turn to our estimates of turnout using our two measures of heterogeneity.

Table 3: Examples of studies that find a negative effect of community size on participation

Author(s)	Type of Participation	Variable
Alesina et al. (1999)	Financing public expenditure on roads	$\log(N)$
Alesina and La Ferrara (2000)	Membership in political, religious, recreational groups, etc.	$\log(N)$
La Ferrara (2002)	Membership in social and economic groups	$\log(N)$
Rose (2002)	Attendance at town meetings, signing petitions, contacting politicians	$\log(N)$
Costa and Kahn (2003b)	Non-desertion from the Union Army during American Civil War	$\log(N)$[1]
Campbell (2005)	Attendance at town or school meetings	N

[1] N measures the size of the place of enlistment.

5 The Turnout Regression Revisited

The above discussion implies the inclusion of the probability that a single vote is decisive as given by equations (2) and (3), the Herfindahl-Hirschman index of linguistic heterogeneity, and $\log(N)$ as a measure of a community's heterogeneity in an empirical model in order to explain voter turnout. Unfortunately, the theoretical discussion does not provide a functional form for such model. As before, therefore, we present the results for both a linear specification of the model and for one in which logs are taken of all variables. The results appear in Table 4. To allow for the possibility that the parents of children in the schools are more homogeneous as a group than the electorate as a whole, we have included a dummy variable to control for those cases in which only the parents could vote, in addition to a dummy variable for the referendum type.

The coefficients of all five independent variables in the linear specification have the expected signs and explain roughly one half of the variation in turnout. The explicit measure of decisiveness of a single vote is highly significant, confirming the results previously obtained when this variable was included in the model without the two measures of heterogeneity (Table 2). An F-test with $F(1, 186) = 0.02$ for the OLS and $F(1, 186) = 0.13$ for the conditional median regression shows that the coefficient on the decisiveness variable is insignificantly different from unity, which was not the case when the two heterogeneity measures were not included. Thus, considering this alone, the results now imply that turnout should approach 100 percent as the probability of a single vote being decisive approaches 1.0. Since our measure of the decisiveness of a single vote includes the size of the electorate and thus controls for this effect of N, the negative and significant coefficient on $\log(N)$ must capture some other determinant of turnout. Our theoretical discussion implies that $\log(N)$ is related to the degree of heterogeneity of a community, and that turnout declines as heterogeneity increases. Our second measure of heterogeneity, the Herfindahl-Hirschman measure of linguistic fragmentation, also has a significant negative effect on participation. This result is of particular interest because, as we demonstrated above, the Herfindahl-Hirschman index is highly correlated with various measures of closeness. If we thought of it as a measure of closeness, and thus a proxy for the probability of a vote being decisive, we would expect a *positive* coefficient on the index. The probability of a vote being decisive has been explicitly included in the equation, however, and so the Herfindahl-Hirschman index is freed to capture linguistic heterogeneity and picks up the expected *negative* coefficient for a measure of heterogeneity. The negative coefficient on the Herfindahl-Hirschman index also suggests that mobilization efforts were not an important, omitted factor in these referendums.

If they were, and expected closeness led to greater mobilization as assumed in the literature, then the Herfindahl-Hirschman index should have proxied for mobilization and have picked up a positive coefficient. Note, also, that although the Herfindahl-Hirschman index computed using the split of votes is itself a valid measure of linguistic heterogeneity, the index included in the regression is computed using a census of the actual Bokmål or Nynorsk users reported in Table 1.

Table 4: Turnout regression from the angle of heterogeneity

Dependent Variable	TURNOUT		Dependent Variable	log(TURNOUT)	
Regression Type	OLS[1]	Median[2]	Regression Type	OLS[1]	Median[2]
SEMI-BINDING = 1	0.09	0.16	SEMI-BINDING = 1	0.25	0.39
	(2.31)	(3.4)		(3.56)	(5.81)
	**	***		***	***
PARENTS = 1	0.02	0.10	PARENTS = 1	0.21	0.35
	(0.54)	(1.72)		(2.76)	(3.53)
		*		***	***
P(DECISIVE BALLOT)	1.06	0.82	LOG(P(DECISIVE BALLOT))	0.01	0.01
	(2.65)	(1.67)		(5.25)	(3.24)
	***	*		***	***
HH INDEX	-0.22	-0.15	LOG(HH INDEX)	-0.04	-0.03
	(-2.94)	(-1.79)		(-2.2)	(-1.81)
	***	*		**	*
LOG(ELECTORATE)	-0.10	-0.08	LOG(LOG(ELECTORATE))	-0.43	-0.21
	(-5.36)	(-3.03)		(-2.66)	(-1.84)
	***	***		***	*
CONSTANT	1.19	1.04	CONSTANT	0.16	-0.24
	(9.6)	(6.04)		(0.55)	(-1.06)
	***	***			
ELASTICITY W.R.T. ELECTORATE	-0.20	-0.17	ELASTICITY W.R.T. ELECTORATE	-0.17	-0.12
OBS.	232	232	OBS.	217	217
R^2	0.46	0.30	R^2	0.44	0.28

[1]Robust Standard Errors; [2]Bootstrap Standard Errors, Pseudo R^2;
Numbers in parenthesis are t-statistics; *** 1 percent; ** 5 percent; * 10 percent level of significance.

In the second equation in Table 4, the logarithm of turnout is regressed on the logarithms of the three key variables. The coefficients on these variables again turn out to be of the predicted signs and statistically significant. In both cases the conditional median regression also supported our main hypotheses.

In the second equation in Table 4, the logarithm of turnout is regressed on the logarithms of the three key variables. The coefficients on these variables again turn out to be of the predicted signs and statistically significant. The log-linear specification can be viewed as an approximation of more general nonlinear functional forms, and is a useful baseline model as such. Taking the logarithm has the additional benefit of reducing the skewness in the distribution of the dependent variable (turnout) apparent from the left panel of Figure 1. In both cases the conditional median regression also supported our main hypotheses.

6 Summary and Conclusions

We began this article by discussing two paradoxes in the literature: why do rational people vote in large electorates when the probability that their vote is decisive is close to zero, and why are crude proxies for the probability that a vote is decisive, such as the size of the electorate and the expected closeness of the outcome, significantly related to voter turnouts even though these probabilities must lie near zero. As an answer to the second paradox, we have argued that all the usual measures of closeness including the size of the electorate, or more precisely the logarithm of size, are not only related to the probability that a vote is decisive, but also to the degree of heterogeneity in the community. A growing empirical literature on social capital shows that participation is lower in more heterogeneous communities, and large communities are on average more heterogeneous than small ones. As the size of an electorate grows, the probability of a vote being decisive becomes vanishingly small and these variables should not be capturing this effect. However, even in large communities, they remain valid measures of heterogeneity, and we maintain that this is why they often are significant in turnout regressions.

To test this hypothesis we needed to eliminate the possibility that the Herfindahl-Hirschman index and the logarithm of the size of the electorate would capture the probability of a decisive vote. We did this by including an explicit measure of this probability, and estimated the model for a sample of communities in which these probabilities differ meaningfully from zero. Our results indicate that community size and linguistic heterogeneity have significant negative impacts on voter turnouts, even after controlling for the probability that a single vote is decisive. These variables must capture other factors affecting voter turnouts than just the decisiveness of a vote. We claim that they measure different dimensions of heterogeneity in a community. Our findings challenge the adequacy of the turnout regression based on closeness and size in testing the rational voter hypothesis, as neither a positive correlation between closeness and turnout nor a negative correlation between size and turnout can be exclusively attributed to instrumental voting. This caveat is important given the widespread practice of separating the probabilistic effect of closeness from that of size in turnout regressions.

We thus claim to have resolved the second paradox. We cannot claim, of course, to have resolved the first one. In large electorates, the probability of a vote being decisive is near zero and some other explanation for why people vote than that given by the rational voter hypothesis is required. We believe that our hypothesis on the effects of heterogeneity on turnouts is consistent with most, if not all, of the other explanations that have been offered in the literature. An individual's sense of civic duty, her willingness to behave ethically with respect to her fellow citizens and her urge to express herself on public issues are likely to be weaker the more heterogeneous the community in which she lives.

A Appendix: Proof of Proposition 1

Part (a) is trivially obtained by algebraic manipulation leading to

$$Q(s) = -0.5HH(s) + 0.25 \ . \tag{11}$$

To show Part (b), we obtain an explicit expression for the coefficient of correlation between $HH(s)$ and $A(s)$. Given $HH(s) = 0.5 - 2A(s)^2$ this can be done in terms of $A(s)$, as

$$\rho(HH, A) = \frac{\mathsf{E}A^2\mathsf{E}A - \mathsf{E}A^3}{\sqrt{\mathsf{E}A^2 - (\mathsf{E}A)^2} \cdot \sqrt{\mathsf{E}A^4 - (\mathsf{E}A^2)^2}} \ . \tag{12}$$

While maintaining $p = 0.5$, we now invoke the Central Limit Theorem to approximate the distribution of $s - 0.5$ by $N(0, 1/4N)$. In our empirical application, $s = S_N/N$ gives the fraction of an electorate of size N that casts S_N ballots in favor of Nynorsk. Substituting the following well-known expressions for the first four moments of the normal distribution $N(0, \sigma^2)$

$$\mathsf{E}|N(0,\sigma^2)| = \sigma\sqrt{2/\pi} \ , \tag{13}$$

$$\mathsf{E}N(0,\sigma^2)^2 = \sigma^2 \ , \tag{14}$$

$$\mathsf{E}|N(0,\sigma^2)|^3 = 2\sigma^3\sqrt{2/\pi} \ , \tag{15}$$

$$\mathsf{E}N(0,\sigma^2)^4 = 3\sigma^4 \tag{16}$$

into equation (12) yields the required

$$\rho(HH, A) = -1/\sqrt{\pi - 2} \approx -0.936 \ . \tag{17}$$

Part (c) can be obtained by a change of variable $x = s - 0.5$, followed by algebraic manipulation leading to $En(x) = (0.5 + x)\log(1 + 2x) + (0.5 - x)\log(1 - 2x) - \log(2)$. Applying the linear approximation $\log(1 + x) = x + 0.5x^2 + o(x^2)$ and then changing back to s yields

$$En(s) \approx 3HH(s) - \log(2) + 1.5 \quad \text{for} \quad s \approx 0.5\ . \tag{18}$$

Q.E.D.

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