A Test of the Marginalist Defense of the Rational Voter Hypothesis Using Quantile Regression

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Abstract

This paper uses quantile regression to uncover variations in the strength of the relationship between the expected closeness of the outcome, size of the electorate and voter turnout in Norwegian school language referendums. Referendums with a low turnout show a weak positive effect of closeness and a strong negative effect of size, the opposite being true of referendums with a high turnout. The results substantiate the marginalist defense of the Downsian rational voter hypothesis, which asserts that, while closeness and size cannot explain the absolute level of turnout, they can account for change at the margin.

Key words: rational voter theory, voter turnout, quantile regression
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1 Introduction

The rational voter hypothesis, initially formulated by Downs (1957) and subsequently extended by Riker and Ordeshook (1968), holds that people vote as to maximize the expected individual net benefit of voting. In the individual calculus of voting, the gains from the desired election outcome must be factored by the probability that the vote will be instrumental in bringing about this outcome. This will be the case if the vote creates or breaks an exact tie. Because the probability of this occurring is close to zero in all but the smallest of electorates, rational choice alone cannot adequately explain why so many people routinely choose to cast an ineffectual but costly vote. This is known as the paradox of voting, one of the most persistent puzzles facing the public choice theorist.

The probability of a single vote deciding the outcome of an election rises with the expected closeness of the outcome and falls with the total number of votes cast (Section 2, Appendix A). The rational voter hypothesis therefore predicts voter turnout to be higher in small-scale elections with close outcomes. Matsusaka and Palda (1993), Blais (2000, ch. 1) and Mueller (2003, ch. 14.2) review numerous empirical tests of this prediction, covering a wide range of countries and elections. Despite persistent differences across countries, types of elections and electoral systems, the evidence on the impact of closeness and electoral size on voter turnout is inconclusive, especially with respect to the size.

Grofman (1993) and Blais (2000) argue that the empirical relevance of the rational voter hypothesis can only be salvaged by reducing its claim. Grofman's argument has become known as the marginalist defense of the rational voter hypothesis. The point is that, even if the expected closeness of the outcome and the number of voters cannot predict the level of voter turnout, they can provide an idea of whether and how it is affected by a change in these variables. Grofman's argument draws on the correct interpretation of the existing empirical evidence, which relies on estimates of turnout regressions discussed in Section 2. A turnout regression can only deliver the marginal effect of the explanatory variables (closeness and size) on the dependent variable (turnout). Because the rational voter theory can only explain the marginal effect of closeness and size on voter turnout, one must take a closer look at the strength of this relationship. Grofman proposes estimating a dynamic specification, in which the change in turnout is regressed on the change in closeness and size. In this paper I propose a different approach, in which the static specification is augmented by the more sophisticated technique of quantile regression.

Quantile regression was proposed by Koenker and Bassett (1978). It has found applications in consumer theory, finance, and environmental studies, and is becoming an increasingly popular alternative to the OLS estimation of conditional mean models.[1] Quantile regression can be used to produce a series of estimates, each for a different quantile of the conditional turnout distribution (conditioned on closeness and size). If the election with turnout τ is, say, in the tenth quantile of the turnout distribution, then ninety percent of elections in the sample have turnouts higher than τ. Since lower quantiles correspond to elections with lower turnouts, we can distinguish the impact of closeness and size in elections where turnout was high from those where it was low. Differences in the sensitivity of turnout to closeness and size convey the importance of instrumental motivations in the respective electorates. By allowing to go beyond the conditional mean effect to uncover the impact of closeness and size on the shape of the conditional turnout distribution, quantile regression can deliver results stronger than can possibly be obtained using the OLS regressions in existing empirical studies.

How much predictive power can we expect from the rational choice theory? Perhaps not very much, if we accept the possibility that rational people might vote for reasons other than

[1] For comprehensive surveys of recent developments, see Koenker and Hallock (2001) and Koenker (2005).

instrumental ones, for example, to express their preferences or because they consider voting their civil duty.[2] Once we admit the possibility that people may be gratified by the act of voting rather than the outcome, the existence of a relationship between closeness, size and turnout becomes a moot issue. The presence of several voter motivations raises the question of which conditions promote which type of behavior. It may well be the case that small local elections or referendums with a single clear issue are conducive to instrumental voting, whereas large, mass media assisted national elections provide an attractive arena for expressive and ethical voters. Referendums are particularly well-suited for testing the rational voter hypothesis because the typical issue put on a referendum is very specific. This facilitates the judgment of the expected utility associated with the outcome, the outcome itself being less prone to distortions related to political representation, log-rolling and other forms of strategic voting behavior.

The next section reviews the methodology of the turnout regression, which is based on the probability of a single vote deciding a two-way election. Section 3 emphasizes the need for a more differentiated approach to the empirical validation of the rational voter theory and founds the choice of data. Following a brief discussion of these data in Section 4, Section 5 presents quantile regression estimates. The last section offers some concluding remarks.

[2]Books by Brennan and Lomasky (1997), Brennan and Hamlin (2000) and Schuessler (2000) provide extensive accounts of expressive motivations in mass participation. The ethical voter hypothesis was initially proposed by Riker and Ordeshook (1968). A discussion of the importance of ethical motivations can be found in Blais (2000), who provides survey evidence in its favor.

2 The Turnout Regression

Downsian rational voter hypothesis holds that a rational citizen will vote provided the expected change in utility between her preferred and alternative outcome is larger than the cost of voting:[3]

$$P(i\ is\ decisive)\Delta u_i - c_i > 0. \tag{1}$$

The expected change in utility is usually referred to as the B-term. Turnout should increase with the probability of being decisive and with the difference in utility between the alternatives, while it should decrease with the cost of voting. The difference in utility and cost of voting are difficult to measure, which leaves the probability as the key explanatory variable. In Section 4 I argue that in the following analysis the error of omission should be smaller than in national elections typically studied in the literature on voter turnout.

A vote is decisive when it creates or breaks an exact tie. Under the binomial assumption on the distribution of the voting poll, if p is the prior probability that a vote will be cast in favor of the first alternative, then a single vote will decide the election approximately with the probability

$$P_e \approx \frac{2\exp(-2N(p-0.5)^2)}{\sqrt{2\pi N}}. \tag{2}$$

when N is even (subscripts refer to the parity of N).[4] The decisive vote is a tie-maker in the former and a tie-breaker in the latter case. Formula (2) is Stirling's approximation of the exact probability, further simplified for p close to one half (Appendix A). They show that the efficacy of a vote will rise with closeness and fall with the total number of votes cast. Either probability

[3] See Downs (1957, chs. 11-14) and, for further developments, Tullock (1967, pp. 110-114) and Riker and Ordeshook (1968).

[4] For an odd N, the analogous probability is obtained by replacing N with $N-1$ (Appendix A).

is the highest at $p = 0.5$, or when the expected outcome is a tie and falls rapidly as p diverges from one half. The term $(p-0.5)^2$ is an objective measure of closeness, which I will refer to as the *quadratic measure*. It is an *ex post* measure that can only be justified by assuming rational expectations on the part of voters. If voters are in fact rational, their subjective probability forecasts should be, on average, correct, so that an objective *ex post* measure of closeness would be equivalent to its *ex ante* counterpart. In empirical applications p is represented by the actual *split* of the voting poll. Note that applied literature traditionally assumes sincere voting. A vote is sincere if it truthfully reflects the voter's preferences. Studies on the effect of informational asymmetries on voting behavior in juries show that sincere voting is not rational and cannot be an equilibrium behavior in general (Feddersen and Pesendorfer 1996).

Taking the natural logarithm of equation (2) leads to the following turnout regression:

$$\log(Turnout) = \beta_0 + \beta_1(p-0.5)^2 + \beta_2\log(N) + \beta_3(p-0.5)^2N + \epsilon. \tag{3}$$

The empirical literature knows several variations to the above specifications. The above equation is typically estimated less the interaction term $(p-0.5)^2N$. Although the quadratic measure of closeness is the only measure consistent with the probability (2), two alternative measures of closeness are frequently used in empirical literature: the *absolute value* $|p-0.5|$ and the *entropy measure* $-p\log(p)-(1-p)\log(1-p)$ proposed by Kirchgässner and Schimmelpfennig (1992). Compared to the quadratic measure, the absolute value puts more moderate weight on p's that are far from one half. The entropy measure is a positive and convex function of p. It attains a unique maximum at one half, around which the function is symmetric. This measure differs from the other two in terms of the sign of its effect on turnout, which is positive. The expected signs on the coefficients are $\beta_1, \beta_2, \beta_3 < 0$ for the quadratic and the absolute value measures, but $\beta_1 > 0$ and $\beta_2, \beta_3 < 0$ for the entropy measure. One advantage of the entropy measure is that it can be generalized in such a way as to be applicable to an election with more than two alternatives. Note that, unlike the former two measures, the entropy measure is not defined for $p = 0$ or $p = 1$, i.e. when the expected outcome is unanimous.

Two further points are worth noting. First, the above specifications imply an inverted U-shape relationship between voter turnout and split about the point $p = 0.5$, in which the probability of being decisive attains its maximum. This relationship can be tested using the following slightly more general specification

$$\log(Turnout) = \beta_0 + \beta_{11}p + \beta_{12}p^2 + \beta_2 \log(N) + \beta_3(p - 0.5)^2 N + \epsilon. \tag{4}$$

Here we expect $\beta_{11} = -\beta_{12}$, $\beta_{11} > 0$. Second, neither specifications can be used to forecast turnouts, as the dependent variable is not constrained to the unit interval. The common way to address this problem is to apply the logistic transformation to the dependent variable: $\log(Turnout/(1-Turnout))$. Unfortunately, unlike the log-linear Downsian model, the resulting specification is highly nonlinear. In Section 5 I test all three measures of closeness, the invested U-shape relationship between turnout and split using the alternative specification (4), and a regression with a transformed dependent variable.

2.1 Quantile Regression on Turnout

When estimated by OLS, specification (3) yields the average marginal effects of closeness on the conditional mean of voter turnout. In a semi-logarithmic specification, the marginal effect will depend on the value of the explanatory variable. The strength of the relationship between closeness (size) and turnout is summarized in the magnitude of the coefficient on that variable.

Quantile regression goes beyond the conditional mean effect to uncover the impact of closeness and size on the shape of the conditional turnout distribution. By comparing the estimates for different quantiles of the conditional turnout distribution, we can differentiate the strength of the impact of closeness and size in the conditionally low and high-turnout elections, thereby exploring the heterogeneity in the relationship. Quadratic regression has several other appealing properties such as robustness against outliers, and higher efficiency for a wide range of non-Gaussian error processes.

The objective function of quantile regression minimizes an asymmetrically weighted sum of absolute deviations, instead of the sum of squared residuals. This, and the fact that the partition into conditional quantiles depends on the entire sample, makes estimating quantile regression not even nearly equivalent to running OLS regressions on subsamples of data. Formally, let $Q_\tau(y_i|x_i) = x_i'\beta_\tau$ denote the τ-th conditional empirical quantile function, then

$$\hat{\beta}_\tau = \arg\min_{\beta_\tau \in \mathbb{R}^k} \left\{ \sum_{i \in \{i | y_i \geq x_i'\beta_\tau\}} \tau |y_i - x_i'\beta_\tau| + \sum_{i \in \{i | y_i < x_i'\beta_\tau\}} (1-\tau)|y_i - x_i'\beta_\tau| \right\}. \quad (5)$$

An estimate is typically found by rewriting the above optimization problem as a linear programming problem and solving it using a modified simplex, or an interior point algorithm (Koenker (2005, ch. 6)).

As is also true of OLS regressions, the quality of inference in quantile regression depends on the number of observations and the number of parameters. In the case of quantile regression, it also depends on how finely we partition the conditional turnout distribution. Choosing a fine partition could mean relying on a few extreme observations when estimating regressions for the tail quantiles. Given the moderate sample size of 232 observations, I estimate specification (3) using quantile regression for the 10, 25, 50 (median), 75, and 90 percent quantiles, and compare them with the conventional OLS counterparts. The more parsimonious variant of the former specification without the interaction term is also tested. Finally, a test of significance of the difference between the 90 and the 10 percent quantiles is performed. Under the i.i.d assumption on the distribution of the error process, the test statistic is asymptotically distributed as χ^2 (Koenker (2005, ch. 3.3.2)).

3 Closeness, Size and Turnout

A great part of the difficulty in validating the Downsian theory using regression analysis lies in the fact that closeness and size reflect phenomena larger and more complex than the efficacy of a vote. Matsusaka and Palda (1993), Kirchgässner and Schulz (2005) and others have argued that closeness indicates the intensity of the electoral competition. Closeness will thus reflect the pressure put on the voters rather than how they perceive the efficacy of their votes.[5] A positive correlation between closeness and turnout therefore does not imply instrumental voting, but rather how well voters are mobilized.

The issue of size is even more problematic. First, different theories of why people vote have generated conflicting predictions with respect to size. Second, and more importantly, the influence of size goes far beyond the probabilistic effect on the decisiveness of a single vote. The following examples should serve to illustrate some facets of this highly complex relationship. Schuessler (2000) imputes voters with both instrumental and expressive motivations. As the expressive voter derives utility from attaching herself to a collective election outcome, her expressive benefit will be proportional to the size of the collective to which she belongs. This results in a non-monotonic relationship between size and turnout, as large electorates confer potentially large expressive benefits, but strip the vote of all power. Schuessler's theory thus

[5] See also Aldrich (1995), Aldrich (1997) and Schachar and Nalebuff (1999).

offers an explanation of why the presence of expressive motivations may be responsible for the lack of definitive empirical evidence with respect to size. Another example is the non-selfish voter theory by Edlin, Gelman and Kaplan (2005). If voters have social preferences and care about the well-being of other citizens, then the expected utility of voting could be approximately independent of the size of the electorate. This is because the subjective utility associated with imposing the desired election outcome on others, while being proportional to the size of the community, is balanced by the probability that the vote is decisive.

Finally, some explanations do not assign voters any specific motivations. Barry (1970) and Aldrich (1995), for example, argue that the expected benefits and costs of voting are simply too small for the calculus of voting to be a meaningful behavioral postulate. This view is often accompanied by the claim that most voters routinely misjudge and even ignore the efficacy of their vote. The survey evidence reported in Blais (2000) to an extent corroborates this view.

It seems that at least part of the difficulty in obtaining definitive empirical evidence on the rational voter hypothesis lies in the roundabout approach taken in the literature. A direct calculation using formula (2) shows that in an electorate of just 1001 voters the probability that a single vote will be decisive cannot exceed 0.0252. The numerical smallness of the direct probability measure poses a great empirical difficulty. As the discussion in the previous section indicated, a common way of circumventing this problem is to separate the probabilistic effect of closeness from that of size. One drawback of doing this is that, taken separately, closeness and size will pick up effects quite unrelated to the efficacy of the vote. The larger and more significant the election, the more distorted the relationship between closeness, size, and turnout are likely to be. Using data for voter turnouts in Norwegian school language referendums, Kaniovski and Mueller (2006) have tested an alternative explanation of why the size of the electorate may reduce turnout. Large communities are, on average, more heterogeneous. From the literature on community participation surveyed in Costa and Kahn (2003) we know that the willingness to participate decreases with heterogeneity. The detrimental effect of heterogeneity on participation in general, and turnout in particular, may compound the probabilistic effect of size on the decisiveness of the vote. It therefore comes as no surprise that the size of the electorate has little explanatory power in large elections, such as national presidential or legislative elections, or in countrywide referendums. The larger the electorate, the more distorted we believe the relationship between closeness, size, and turnout will be. This must be especially true with respect to size.

4 The Data

For closeness and size to have a reasonable explanatory power, we need to turn our attention to small electorates, in which pronounced instrumental motivations can realistically be expected. Furthermore, the majority of empirical studies derive specifications based on the probability that one vote will decide an election with only two alternatives discussed in Section 2. Both considerations point to local referendums as the best source of data for testing the rational voter hypothesis. The turnout record in 232 school district referendums in Norway ideally fulfills the smallness and the binary choice criteria, and has already been used in Søberg and Tangerås (2004) to test the rational voter hypothesis, as well as in Kaniovski and Mueller (2006) to study the effect of heterogeneity on voter turnout.

On 232 occasions between 1971 and 2003, Norwegians were asked which of the two official languages, Bokmål or Nynorsk, should be the primary language of their school district. With relatively small electorate sizes, ranging from 6 to 4,625 and an average of 395 voters, these referendums fulfill the smallness criterion while still offering sufficient variability for robust empirical inference, covering more than three decades and 76 municipalities in 13 of Norway's 19 counties (Table 1).

TABLE 1 ABOUT HERE

Søberg and Tangerås (2004) estimate an OLS regression using the absolute value as the measure of closeness. They find both the size of electorate in the school district and the expected closeness of the outcome to be good predictors of voter turnout. Prior to 1985, the referendums were semi-binding (binding, provided that at least 40 percent of the electorate voted in favor). All 86 referendums since 14.06.1985 have only been advisory, although the outcomes of all except four of the advisory referendums were implemented by the municipal authorities. Participation in some referendums was limited to the parents of school children.[6] Søberg and Tangerås (2004) show that both circumstances have had their predicted effect on voter turnout, which has also been confirmed by Kaniovski and Mueller (2006). Higher turnouts in semi-binding referendums reflect the fact that a vote is more decisive in this type of referendum. Extending the franchise to parents only further increased turnout, after controlling for the size effect, presumably because the parents of school children were more concerned with the issue than the general public. These previous findings illustrates the importance of subjective utility derived from the desired election outcome in these referendums, which is the B-term in the Downsian model.

Consistent with the expected utility maximization, the decision to vote will depend on this utility, which is the B-term in the Downsian model. Although it is virtually impossible to capture the B-term empirically, we shall expect that any collective outcome will bring different subjective utilities to different people. This heterogeneity will rise with the size of the community and the number and complexity of the issues, and could well be responsible for the empirical difficulties mentioned in the introduction.

[6]This was the case from 01.07.1971 to 31.07.1985, and from 01.08.1999 to 31.07.2000.

The relative simplicity of the issue at hand suggests that voters derive roughly similar subjective utilities, which can therefore be omitted from the analysis. In a series of national legislative elections where several parties are pushing a variety of issues - some openly, others covertly, the heterogeneity of subjective utilities is likely to be much higher than in a series of school language referendums. The above considerations make Norwegian school-district referendums an attractive choice for testing the rational voter hypothesis.

5 Quantile Regression Results

I begin by estimating quantile regressions for the three specifications - one for each measure of closeness - for the 10, 25, 50, 75, and 90 percent quantiles of the conditional turnout distribution. To control for the fact that a vote in a semi-binding referendum is more decisive than in an advisory referendum, a dummy variable discriminating between the two legal settings is included. I do not control for “parents only” ruling, as its effect on decisiveness is already reflected in the smaller size of the electorate, and I am primarily concerned with the precise measurement of the effect of closeness and size. Table 2 compares quantile regression estimates to the corresponding OLS estimates contained in the second column.

Results reported in Table 2 show that the coefficients on closeness and size have their expected signs in all regression and are mostly significantly different from zero, all variables together explaining between 20 and 60 percent of the observed variation in turnout. The factors that increase the efficacy of the vote also increase voter turnout. This would be our conclusion even if we were confined to OLS (the second column in Table 2). It is equally apparent, however, that the OLS regression obscures important detail. First, the effects of closeness and size vary for different portions (quantiles) of the conditional turnout distribution and, second, this variation has a pattern. Since lower conditional quantiles correspond to the referendums with lower turnouts, the results show that low-turnout referendums have had a weaker positive impact of closeness and a stronger negative impact of the electorate size, the opposite being true of high-turnout referendums. In other words, the disparity between turnouts in close and clear-cut referendums is substantial, particularly at the left tail of the conditional distribution, and this disparity decreases nearly monotonically as we move to the right tail of the distribution. A similar statement can also be made for the size. This further substantiates the prediction of the Downsian model.

The differences in the sensitivity of turnout to closeness and size increase nearly monotonically across quantiles. To test whether the differences in the strength of the relationship between closeness, size and turnout are statistically significant, I test for the difference in the 10 and 90 percent quantiles estimates. The last column in Table 2 reports the magnitude of these differences, which are indeed significant. Not only does this test confirm the predicted effect of closeness and size, it also shows these variables to cause the dispersion in observed turnout levels. Interestingly, the coefficients on the dummy variable do not show a monotonic pattern, despite the fact that most coefficients are highly significant and have their predicted signs. The type of the referendum has had a more uniform effect on voter turnout in these referendums.

As a further test, I estimate a more parsimonious specification, one without the interaction term and the dummy variable. This time I estimate quantile regression for the 10, 20, . . . , 90 percent quantiles (the deciles) for the three specifications. Figure 1 plots the coefficients on closeness and size for the deciles of the conditional turnout distribution. Estimates for the consecutive quantiles and their 95 percent confidence intervals are connected by solid lines. To facilitate comparison, the horizontal axis is centered on the OLS estimate, its 95 confidence interval shown with hatched lines. All coefficients have their expected signs and are significantly different from zero in every regression. Results of the parsimonious specification corroborate those reported in Table 2, sowing a surprisingly clear nearly monotonic pattern. A lower observed turnout is again accompanied by a weaker positive effect of closeness and a stronger negative effect of size, all results being robust to the choice of closeness measure.

The results from the two alternative specifications presented in Table 3 closely resemble those of the basic specification with the quadratic measure of closeness at the top of Table 2. When entered separately, *Split* and $Split^2$ produce estimates of opposite signs and similar absolute values. With $F(1, 226) = 0.29$ for the first $F(1, 221) = 0.36$ for the second OLS specification, a Wald-test of the linear restriction $\beta_{11} + \beta_{12} = 0$ indicates no difference in the absolute values of the two coefficients. This is also true for the individual quantile estimates. For example, the test statistics for the 10 percent quantile estimates are, respectively, $F(1, 226) = 1.60$ and $F(1, 221) = 0.42$. In sum, alternative specifications indicate the robustness of the basic turnout regression and the existence of an inverted U-shape relationship between voter turnout and split, as predicted by the Downsian model.

6 Summary

The marginalist defense of the Downsian rational voter hypothesis asserts that, while closeness and size cannot explain the absolute level of turnout, they can account for change in these variables. In this paper I show that a regression analysis more sophisticated than that hitherto employed in the literature can add further weight to the marginalist cause.

The novelty of this study lies in its use of quantile regression to investigate the heterogeneity in the strength of the relationship between closeness, size and turnout. Quantile regression reveals the impact of closeness and size on the shape of the conditional turnout distribution and thus delivers results stronger than can possibly be obtained using OLS regressions in existing empirical studies.

Survey evidence tells us that voters are driven by several distinct motivations and this urges us to consider which conditions promote which type of behavior. It seems reasonable that large, mass media assisted national elections may be an attractive arena for the expressive and the ethical voter, while small local elections or referendums with a single clear-cut issue may be more conducive to instrumental voting.

The empirical results presented in this paper support the second hypothesis. Whatever caused the differences in turnout in the 232 Norwegian school language referendums, they can to a large extent be explained by factors relating to instrumental voting. Quantile regression shows that a lower observed turnout is accompanied by a weaker positive effect of closeness and a stronger negative effect of size, with the differences being significant and robust to the choice of closeness measure. This pattern corroborates the average marginal effect uncovered by OLS. Both findings support the marginalist defense.

A Probability that a vote is decisive in a two-way election

When a voter faces two alternatives her vote becomes decisive either when all other votes would have tied the outcome (Event 1), or when her preferred alternative would lose by a single vote if she abstained (Event 2). The two events are mutually exclusive, as N is odd in the former case and even in the latter. Let p be a prior probability of a vote being cast for the voter's preferred alternative. Event 1 occurs with probability P_o, which is the probability of $\frac{N-1}{2}$ successes in $N-1$ Bernoulli trials with the probability of success p:

$$P_o = \frac{(N-1)!}{(\frac{N-1}{2}!)^2} p^{\frac{N-1}{2}} (1-p)^{\frac{N-1}{2}}. \tag{6}$$

Since N is odd, substitute $N = 2k+1$ for $k = 0, 1, \ldots$

$$P_o = \frac{(2k)!}{(k!)^2} p^k (1-p)^k. \tag{7}$$

By the Stirling's approximation $x! \cong \sqrt{2\pi}(x^{x+0.5}e^{-x})$, where $\cong$ means that the ratio of the right hand side to the left hand side approaches unity as $x \to \infty$,

$$P_o \cong \frac{\sqrt{2\pi}(2k)^{2k+0.5}e^{-2k}}{(2\pi)(k^{k+0.5}e^{-k})^2} p^k (1-p)^k = \frac{2^{2k+0.5}}{\sqrt{2\pi k}} p^k (1-p)^k. \tag{8}$$

Substituting back $k = (N-1)/2$ yields, after some simplification,

$$P_o \approx \frac{2[1-(2p-1)^2]^{\frac{N-1}{2}}}{2\sqrt{\pi(N-1)}}. \tag{9}$$

Note that in $[0,1]$ both $x(1-x)$ and $1-(2x-1)^2$ attain their maxima at $x = 0.5$, so that the approximation preserves P_o's essential property of being highest at $p = 0.5$. Using the fact that

$1 + x \approx e^x$ for small $|x|$ and $1 - (2p-1)^2 \approx e^{-(2p-1)^2} = e^{-4(p-0.5)^2}$, for all p close to 0.5 the above expression can written as

$$P_o \approx \frac{2e^{-2(N-1)(p-0.5)^2}}{\sqrt{2\pi(N-1)}}. \tag{10}$$

This formula leads to the convenient log-linear specification with an interaction term between the quadratic measure of closeness $(p-0.5)^2$ and size N.

Event 2 occurs with probability P_e, which is the probability of $\frac{N}{2}$ successes in $N-1$ Bernoulli trials with the probability of success p. By a similar argument using the parity of N, for all p close to 0.5,

$$P_e \approx \frac{2e^{-2N(p-0.5)^2}}{\sqrt{2\pi N}}. \tag{11}$$

Good and Mayer (1975) discuss the magnitude of error in P_o and P_e due to p deviating from 0.5, which can be substantial (Figure 2). See, also Chamberlain and Rothschild (1981), and in the context of voting power, Grofman (1981). Kaniovski (2008) computes the probability of casting a decisive vote when votes are neither equally probable to be for or against, nor independent. Departures from either assumption induce a substantial bias in this probability compared to the baseline case of equally probable and independent votes. The bias incurred by the probability deviating from one-half is larger than that incurred by the Pearson product-moment correlation coefficient deviating from zero.

FIGURE 2 ABOUT HERE

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Table 1: Descriptive statistics

	Obs.	Mean	St.Dev.	Min.	Max.
Turnout	232	0.66	0.23	0.10	1.00
Electorate	232	394.98	561.48	6	4625
Split	232	0.47	0.18	0	0.89
	MEASURES OF CLOSENESS				
Quadratic Measure	232	0.03	0.05	0	0.25
Absolute Value	232	0.14	0.12	0	0.50
Entropy Measure	229	0.63	0.11	0.05	0.69

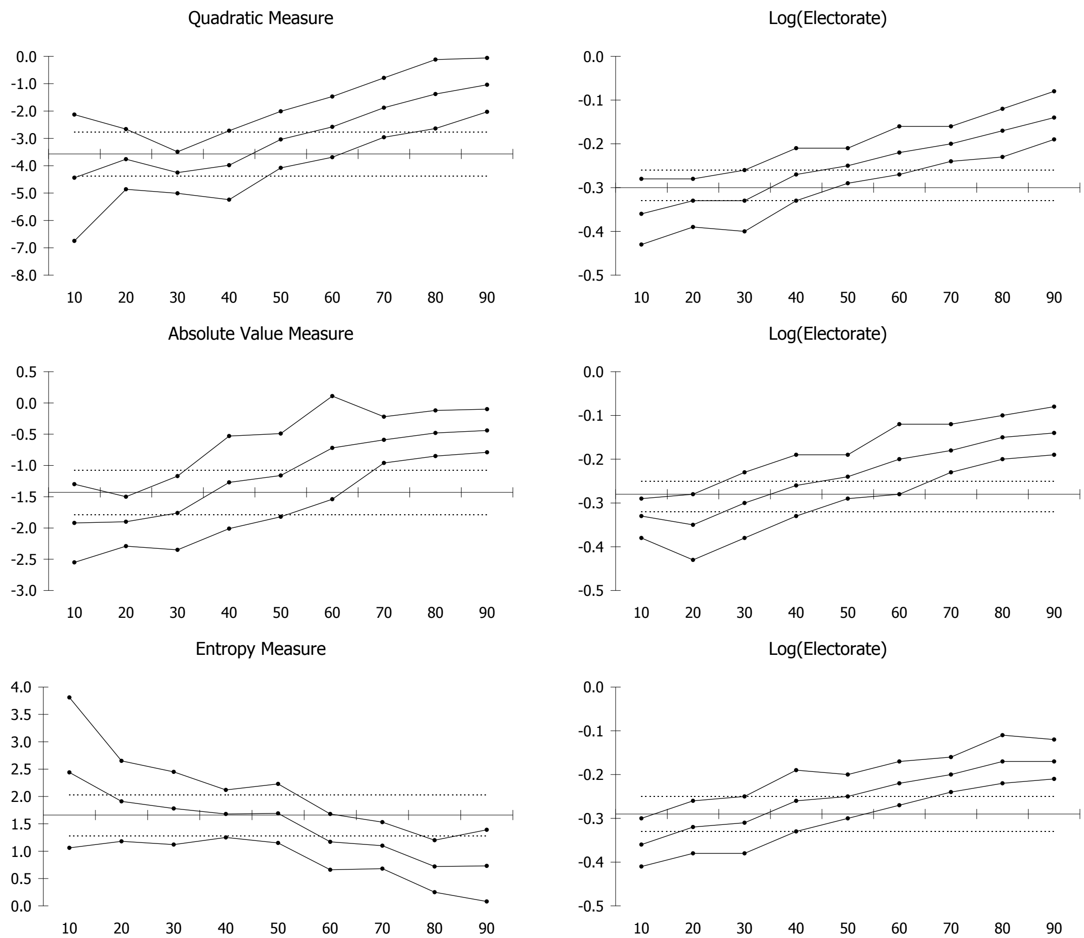
The split is defined as the ration of votes in favor of Nynorsk to the total number of votes cast. The entropy measure is not defined for unanimous outcomes, hence fewer observations.

Table 2: Quantile regression

Quantile in percent[1]	OLS[2]	10	25	50	75	90	90-10
Dependent Variable				Log(Turnout)			
Quadratic Measure	-2.226	-3.266	-2.755	-1.225	-0.508	-0.821	2.445
	(-4.26)	(-2.70)	(-3.18)	(-1.85)	(-0.93)	(-1.51)	(1.93)
	***	***	***	*			*
Log(Electorate)	-0.206	-0.276	-0.202	-0.135	-0.100	-0.103	0.172
	(-6.84)	(-4.58)	(-3.35)	(-5.40)	(-3.79)	(-4.42)	(2.77)
	***	***	***	***	***	***	***
Inter. Term	-0.004	-0.006	-0.005	-0.007	-0.007	-0.001	0.004
	(-3.60)	(-2.12)	(-2.25)	(-4.05)	(-2.62)	(-0.45)	(1.11)
	***	**	**	***	***		
Semi-binding = 1	0.132	0.181	0.281	0.180	0.159	0.108	-0.074
	(2.49)	(1.10)	(2.20)	(3.44)	(3.23)	(3.37)	(-0.45)
	**		**	***	***	***	
R^2	0.59	0.45	0.42	0.36	0.27	0.23	
Absolute Value	-0.908	-1.668	-1.244	-0.387	-0.065	0.015	1.682
	(-4.13)	(-4.92)	(-3.13)	(-1.61)	(-0.36)	(0.09)	(4.55)
	***	***	***				***
Log(Electorate)	-0.169	-0.280	-0.191	-0.100	-0.070	-0.057	0.224
	(-6.08)	(-4.81)	(-3.50)	(-3.91)	(-3.19)	(-2.37)	(3.69)
	***	***	***	***	***	**	***
Inter. Term	-0.001	-0.001	-0.002	-0.002	-0.002	-0.002	-0.001
	(-4.78)	(-1.30)	(-3.04)	(-3.52)	(-3.05)	(-2.72)	(-1.10)
	***		***	***	***	***	
Semi-binding = 1	0.125	0.133	0.175	0.204	0.140	0.061	-0.072
	(2.37)	(0.95)	(1.62)	(4.20)	(2.76)	(1.89)	(-0.53)
	**			***	***	*	
R^2	0.58	0.46	0.42	0.34	0.26	0.24	
Entropy Measure	1.754	2.802	2.288	1.732	0.702	0.435	-2.367
	(8.56)	(4.78)	(6.26)	(5.21)	(1.86)	(2.38)	(-3.94)
	***	***	***	***	*	**	***
Log(Electorate)	-0.174	-0.186	-0.138	-0.063	-0.092	-0.084	0.103
	(-4.16)	(-2.24)	(-2.54)	(-1.60)	(-2.06)	(-2.63)	(1.20)
	***	**	**		**	***	*
Inter. Term	-0.000	-0.000	-0.001	-0.001	-0.000	-0.000	0.000
	(-2.06)	(-1.29)	(-3.91)	(-3.18)	(-1.24)	(-1.10)	(0.63)
	**		***	***			
Semi-binding = 1	0.095	0.100	0.116	0.146	0.107	0.099	-0.001
	(1.81)	(0.69)	(1.23)	(3.33)	(2.07)	(3.03)	(-0.01)
	*			***	**	***	
R^2	0.60	0.46	0.46	0.37	0.27	0.25	

[1]Bootstrap Standard Errors, Pseudo R^2; [2]Robust Standard Errors;
*** 1 percent; ** 5 percent; * 10 percent level of significance; The estimate for the constant term is omitted.

Figure 1: Quantile regression results for the three measures of closeness



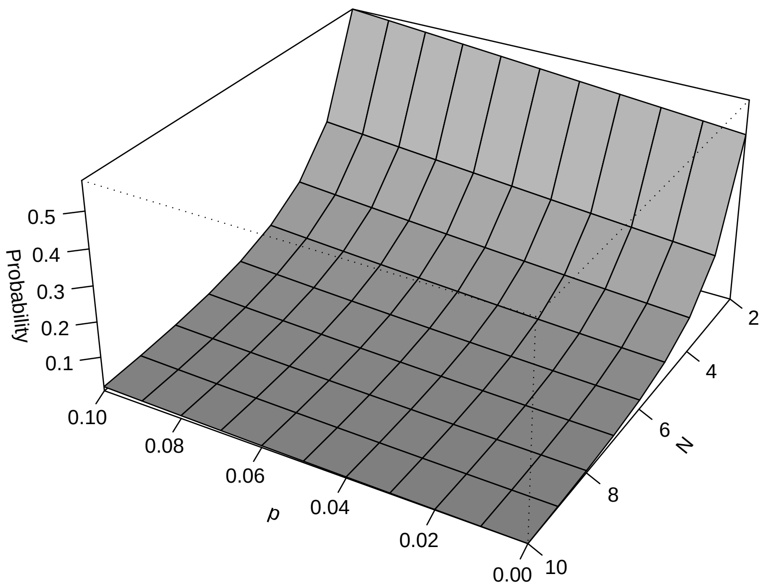
The horizontal axis crosses the vertical axis at the OLS estimate, whose 95 percent confidence interval is indicated with hatched lines.

Table 3: Quantile regression. Alternative Specifications.

Quantile in percent[1]	OLS[2]	10	25	50	75	90	90-10
Dependent Variable				Log(Turnout)			
Split	2.157	4.510	2.939	1.133	0.480	0.817	-3.694
	(4.05)	(3.48)	(3.26)	(1.63)	(0.82)	(1.44)	(-2.67)
	***	***	***				***
Split^2	-2.070	-5.093	-2.996	-1.016	-0.378	-0.648	4.445
	(-3.51)	(-3.21)	(-2.78)	(-1.61)	(-0.62)	(-1.10)	(2.68)
	***	***	***				***
Log(Electorate)	-0.207	-0.291	-0.209	-0.136	-0.105	-0.125	0.165
	(-6.83)	(-5.09)	(-3.51)	(-5.11)	(-4.13)	(-6.16)	(2.73)
	***	***	***	***	***	***	***
Inter. Term	-0.005	-0.004	-0.005	-0.007	-0.007	-0.002	0.002
	(-3.66)	(-1.48)	(-2.18)	(-4.09)	(-2.56)	(-0.63)	(0.64)
	***		**	***	**		
Semi-binding = 1	0.131	0.088	0.281	0.193	0.156	0.082	-0.006
	(2.48)	(0.60)	(2.27)	(3.73)	(3.37)	(3.00)	(-0.04)
	**		**	***	***	***	
R^2	0.59	0.46	0.42	0.36	0.27	0.24	
Dependent Variable				Log(Turnout/(1-Turnout))			
Split	6.973	8.684	9.094	7.911	5.255	4.54	-4.145
	(4.04)	(3.43)	(4.21)	(3.01)	(1.57)	(1.34)	(-1.03)
	***	***	***	***			
Split^2	-6.739	-9.242	-9.100	-7.719	-5.005	-3.891	5.351
	(-3.69)	(-2.97)	(-3.40)	(-2.96)	(-1.49)	(-1.16)	(1.22)
	***	***	***	***			
Log(Electorate)	-0.622	-0.585	-0.608	-0.565	-0.565	-0.766	-0.181
	(-7.91)	(-4.03)	(-4.83)	(-5.31)	(-4.64)	(-8.44)	(-1.13)
	***	***	***	***	***	***	
Inter. Term	-0.003	0.000	-0.002	-0.002	-0.005	0.001	0.001
	(-1.21)	(0.01)	(-0.53)	(-0.54)	(-0.98)	(0.10)	(0.08)
Semi-binding = 1	0.412	0.300	0.489	0.511	0.597	0.560	0.260
	(3.01)	(1.10)	(2.15)	(3.68)	(3.00)	(3.40)	(0.86)
	***		**	***	***	***	
R^2	0.52	0.36	0.38	0.34	0.30	0.31	

[1]Bootstrap Standard Errors, Pseudo R^2; [2]Robust Standard Errors;
*** 1 percent; ** 5 percent; * 10 percent level of significance; The estimate for the constant term is omitted.

Figure 2: Approximation to the probability of casting a decisive vote



In an election with two alternatives, the *Probability* of being decisive decreases with the size of electorate N and with $|p - 0.5|$. For a fixed N, the probability is the highest when $p = 0.5$ and decreases rapidly as p deviates from 0.5. The approximation is valid for $p \approx 0.5$.