Estimating the Probability of Acting as a Trustee

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Abstract

    We discuss a binomial mixture model for estimating the probability of a political representative acting as a delegate or a trustee. The model also returns the probability of congruence of a representative with the national median voter. The estimated probability of congruence strongly correlates with the observed frequency of congruence, which was obtained by matching parliamentary roll-call votes with the will of the median voter revealed in Swiss national referendums on identical legislative proposals. Since our method uses the roll-call votes of political representatives as sole input, it can be used to infer congruence levels of politicians, even if the will of the median voter is unobserved.

Keywords: Political representation, delegate, trustee, binomial mixture model.

JEL-Codes: C13, D72.

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1 Introduction

The delegate vs. trustee behavioral dichotomy is a classical theme in political science. It dates back to Irish statesman and philosopher Edmund Burke's analysis of political representation. In his 1774 speech to the Electors of Bristol, he argued in favor of the trustee model of representation: "Your representative owes you, not his industry only, but his judgement; and he betrays you instead of serving you if he sacrifices it to your opinion" (Burke 1774 (2013)). Simply put, delegates vote the way their constituents want them to vote, whereas trustees exercise their independent judgment to vote for what, in their view, is best in the public interest. This theme has been at the heart of the theory of political representation for more than two centuries, with normative and positive aspects to it.[1]

The normative question of how representatives *should act* is unlikely to be settled by an application of scientific method alone. The investigation into the positive question of how the representatives *act* has generated extensive empirical literature on the strength of the link between the representatives and the voters, as well as the factors that encourage the former to act as a delegate or a trustee.[2] This paper contributes to the empirical literature by proposing a novel empirical approach for estimating the probability that a representative follows the will of the median voter, and the probability that she acts as trustee or as delegate.

Theoretically, many explanations can be advanced for why representatives may act as delegates or trustees (Fox and Shotts 2009). For one, the competence of the representative with respect to the issue on the ballot should matter. A school teacher, for example, serving as an MP might feel competent to exercise independent judgment on education and less inclined to do so on defense. But if the self-assessment is not shared by the electorate, the representative might be better off acting as a delegate rather than a trustee. A potential mismatch between the preferences of the representatives and those of the electorate may also play a role. This line of reasoning is directly relevant to the empirical analysis conducted in this paper, which tests the congruence between a legislator elected by a geographical constituency and the national median voter. The legislator is supposed to represent the whole electorate, while having an incentive to cater to her constituency. How the split loyalties play out in a conflict of interest is an empirical question. Fox and Shotts (2009) argue that the incentive to act as a delegate increases if the representative believes that the electorate is uncertain about her preferences. In this case, the representative has an incentive to gain trust, thus adding leeway for the exercise of independent judgment. Another group of factors includes the democratic institutions

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[1]For a classic reference, see Pitkin (1967). Comprehensive surveys can be found in Przeworski, Stokes and Manin (1999) or Ashworth (2012). A survey of fiscal policy outcomes associated with referendums in US states and Swiss cantons by Matsusaka (2018) shows that, overall, initiatives leads to policies close to those preferred by a majority of citizens.

[2]See, Stokes (2001) and Fox and Shotts (2009). Further examples of empirical studies include Portmann (2014) and Matsusaka (2015). A discussion on citizen-elite ideological congruence can be found in Golder and Ferland (2018).

and the electoral behavior that provides feedback in the form of rewards or punishment of the representative. Do the voters judge the representatives based on their policies or the outcomes of their policies? The importance of policies compels the representative seeking reelection to act as a delegate. When the voters vote retrospectively by judging the outcomes, exercising independent judgment will be rewarded if it leads to a successful policy outcome.

The main implication of the above considerations is that the diversity of ballot issues should induce frequent changes in the mode of representation by the representative, which in turn makes the outcome of an election a very imperfect indicator of whether the representative has acted more like a trustee or a delegate. This is especially true if the issues differ in their salience, and because representatives vote on issues much more frequently than voters vote for the representatives or their parties in general elections.

Identifying the actual mode of representation has been difficult in practice. Whether representatives vote according to their own preferences or the preferences of their constituents cannot be tested directly, unless surveys of public opinion or instruments of direct democracy are used to reveal the latter (Stadelmann, Portmann and Eichenberger 2013, Brunner, Ross and Washington 2013, Barceló 2019). A test of the fidelity of representation in an indirect democracy must remain an indirect one, as the preferences of constituents are usually unobservable. In practice, such tests would have to compare the ideological position of a legislator to that of the median voter (Poole and Rosenthal 1997, Gerber and Lewis 2004, Matsusaka 2010). The ideological position of the legislator follows from her voting record (roll-call votes), whereas the ideological position of the median voter can be estimated from opinion surveys and election results, with techniques ranging from single-dimensional scores to multi-dimensional spatial fixings. But no matter how accurate ex post measures of congruence might be, there will always be doubt borne by observational equivalence regarding the mode of representation. The mere observation of congruence between a representative and her constituency does not imply the delegate mode of representation, as the representative may make a popular choice for *idiosyncratic* reasons. The representative may be congruent with the constituency, while acting as a trustee.

We contribute to the empirical literature on political representation by proposing a method of estimating the degree (technically a probability) to which a representative acts as a delegate or a trustee over a large number of parliamentary bills. Our approach overcomes the problem of observational equivalence between the two modes of representation. Our binomial mixture model of the representatives' votes involves unobservable idiosyncratic factors – two for each representative and a single unobservable common factor. The model is estimated using the roll-call votes of the representatives as sole input. The estimates are then used to compute the probability of a representative acting as a trustee. The second quantity of interest is the probability of a representative voting according to an unobserved common signal that influences the votes of all representatives. This would be the probability of congruence with the national median voter, should the common signal indeed reflect the will of the national median voter. Under this assumption, the model yields an estimate of the fidelity of representation based on roll-call votes alone.

The approach proposed in this paper resonates with some ideas found in the math-

ematical social science literature on the Condorcet Jury Theorem. One problem in the jury setting is how to verify the quality of a decision. Whereas the jurisdiction of a lesser court can be reviewed by a higher judicial authority, the correctness of the jurisdiction by a Supreme Court is unobservable. How can we judge the justices? The older literature on the classical Condorcet Jury Theorem, reviewed in Grofman, Owen and Feld (1983), proposed comparing an individual's vote with the majority vote. The more often an individual is congruent with the majority, the more frequently she is correct. This approach is not entirely satisfactory due to a degree of tautology inherent to it. In contrast, the interpretation of the common signal in this paper is inspired by the analysis of voting in the US Supreme Court (Iaryczower and Shum 2012). The reasoning is as follows: If judges vote according to a common signal, which by exclusion must be the evidence presented, rather than an idiosyncratic signal such as personal ideology, we can be confident that the decision was sound. Similarly, if a legislator votes in parliament according to a factor that influences the votes of *all* legislators, we may trust that her decision will coincide with the will of the national median voter and calculate the probability of congruence.

The key assumption of the common signal representing the will of the national median voter cannot be tested if the latter is not observed, as is usually the case in the practice of political decision-making. The Swiss political system offers a unique design for testing congruence by requiring the representatives to vote in parliament before placing the same issue in a country-wide referendum. All constitutional amendments passed by parliament require a referendum. A small group of citizens can start an initiative to amend the constitution or demand a referendum on laws enacted by parliament (Stadelmann et al. 2013, Portmann 2014, Hessami 2016).

Observing the actual will of the national median voter allows us to *validate* our model. We compare the estimated individual probabilities of congruence with the observed individual frequencies of congruence and find a significant correlation between them. This implies that our estimation method can be applied to identify politicians who are frequently congruent with the median voter based on their roll-call votes alone. The ability to accurately predict the probability of congruence lends validity to the estimated probability of a representative acting as a delegate or a trustee. To provide a further example for the application and usage of the estimated probabilities in empirical work, we use ideology scores for Swiss MPs – computed using the well-known Poole and Rosenthal's (1997) NOMINATE method – to explore the relationship between the ideology and the probability of acting as a trustee on the individual level.

2   The model

In an assembly of n legislators, each legislator i, where $i = 1, 2, \ldots, n$, may vote according to a private binary signal X_i or a common binary signal M. The choice of legislator i is modeled by a binary random variable L_i. Assume that the $2n + 1$ random variables L_i, X_i, M are independent Bernoulli random variables with the expectations $\mathbf{E}L_i = r_i$,

$\mathbf{E}X_i = r_{n+i}$ and $\mathbf{E}M = r_{2n+1}$. The vote V_i is modeled as a mixture:

$$V_i = L_iX_i + (1 - L_i)M. \tag{1}$$

It follows that the distribution of V_i is also Bernoulli, and $\mathbf{E}V_i = r_ir_{n+i} + (1 - r_i)r_{2n+1}$.

The common signal induces positive correlations between the votes, which are only independent conditionally on the common signal. The Pearson product-moment correlation coefficient for any two votes V_i and V_j, where $j = 1, 2, \dots, n$ and $j \neq i$, is given by

$$\mathbf{Corr}(V_i, V_j) = \frac{\mathbf{E}V_iV_j - p_ip_j}{\sqrt{p_i(1 - p_i)p_j(1 - p_j)}} = \frac{(1 - r_i)(1 - r_j)r_{2n+1}(1 - r_{2n+1})}{\sqrt{p_i(1 - p_i)p_j(1 - p_j)}} > 0,$$

where $p_i = \mathbf{E}V_i = r_ir_{n+i} + (1 - r_i)r_{2n+1}$. The votes are also correlated with the common signal

$$\mathbf{Corr}(V_i, M) = \frac{\mathbf{E}V_iM - p_ir_{2n+1}}{\sqrt{p_i(1 - p_i)r_{2n+1}(1 - r_{2n+1})}} = (1 - r_i)\sqrt{\frac{r_{2n+1}(1 - r_{2n+1})}{p_i(1 - p_i)}} > 0.$$

In the absence of absenteeism and abstentions, the roll-call data would comprise nT observations of random variables V_i, where n is the number of legislators and T the number of ballots, indexed by $t = 1, 2, \dots, T$. In the next section, we show how to estimate the vector of model parameters

$$\vec{r} = (r_1, \dots, r_n, r_{n+1}, \dots, r_{2n}, r_{2n+1})$$

from the roll-call votes, without observing the $(2n + 1)T$ realizations of L_i, X_i and M.

Conceptually, trustees exercise discretion by using their judgment and voting according to their conscience, whereas delegates follow the (uncertain) will of the majority. To define such behavior in the confines of our stochastic model of votes, we must envision how an impartial external observer would perceive a trustee's votes on a large set of independent and exogenous issues. For an impartial external observer, voting as a trustee would amount to voting *idiosyncratically*. We thus say that legislator i acts as a trustee on the ballot t, if i votes on t according to the t's realization of i's private and independent signal X_i. This allows us to interpret $\mathbf{E}L_i$ as the probability of acting as a trustee. Our primary objective is to estimate the probability of i acting as a trustee based on i's voting record.

It is important to emphasize that the nature of the common signal M, as a source of stochastic dependence between votes, has no direct bearing on the identification of trustee voting. Ideally, we would like to interpret the complementary probability $(1 - \mathbf{E}L_i)$ as the probability of acting as a delegate, so that the mixing equation (1) determines the choice between the two classic modes of political behavior. This interpretation is only admissible if M indeed reflects the will of the national median voter, an assumption that in most cases cannot be verified if the latter remains unobserved. For a typical indirect democracy, i.e. a parliamentary democracy without referendum decisions, our analysis

would end here, because an indirect identification of the common signal is unlikely, due to many potential factors. The model tells us only that the realizations of M are binary, that they influence the votes of all legislators and are specific to each ballot, but this information alone will rarely suffice to identify M empirically.

In the case of Swiss referendums, we can *validate* the assumption of the common signal M being the will of the national median voter by comparing the predicted probabilities of congruence with the observed frequencies of congruent votes. We use the actual outcome of the referendum as an ex ante estimate of the national median voter's position.

The assumption that the common signal reflects the national median voter's position allows us to predict the probability of congruence of individual politicians with the median voter using the roll-call data. The probability of congruence with the median voter M can be obtained from the following conditional probabilities:

$$\pi_{11} = \mathbf{P}\{V_i = 1 \mid M = 1\} = 1 - r_i(1 - r_{n+i}),$$
$$\pi_{00} = \mathbf{P}\{V_i = 0 \mid M = 0\} = 1 - r_i r_{n+i},$$
$$\pi_{10} = \mathbf{P}\{V_i = 1 \mid M = 0\} = r_i r_{n+i},$$
$$\pi_{01} = \mathbf{P}\{V_i = 0 \mid M = 1\} = r_i(1 - r_{n+i}).$$

The probability of congruence is given by

$$\mathbf{P}\{V_i = M\} = r_{2n+1}\pi_{11} + (1 - r_{2n+1})\pi_{00}. \tag{2}$$

Here, the first term is the probability of congruence of a Yes vote:

$$\mathbf{P}\{V_i = 1 \cap M = 1\} = r_{2n+1}\pi_{11},$$

whereas
$$\mathbf{P}\{V_i = 0 \cap M = 0\} = (1 - r_{2n+1})\pi_{00}$$

is the corresponding probability for a No vote.

The *observational equivalence* regarding trustee or delegate behavior discussed in the introduction follows, because the event $V_i = M$ may occur with any L_i; i.e. a representative acting as a trustee or as a delegate may represent the will of the median voter. Put differently, the mere observation of congruence of a legislator's vote (realization of V_i) with the median voter (realization of M) does not imply a certain mode of representation. A representative may make a popular choice ($V_i = M$) for idiosyncratic reasons ($L_i = 1$). The above model allows overcoming observational equivalence by disentangling the two cases, which is a central new contribution of our approach to the existing literature.

2.1   Maximum likelihood estimation

We can estimate $\vec{r}$ using the method of Maximum Likelihood from the parliamentary roll-call data, i.e. realizations v_i of the random variables V_i, despite X_i, L_i and M being unobserved. Let $v_i = 1$ if legislator i votes Yes, and $v_i = 0$ if i votes No. Let n be a fixed

number of legislators, and let v_i^t be independent (in t) observations of V_i for $t = 1, 2, \ldots, T$ ballots. The likelihood function reads

$$F_T(\vec{r}) = \prod_{t=1}^{T} \left[r_{2n+1} \prod_{i=1}^{n} F(i, M = 1, t, \vec{r}) + (1 - r_{2n+1}) \prod_{i=1}^{n} F(i, M = 0, t, \vec{r}) \right], \qquad (3)$$

where

$$F(i, M = 1, t, \vec{r}) = v_i^t(1 - r_i(1 - r_{n+i})) + (1 - v_i^t)r_i(1 - r_{n+i}),$$
$$F(i, M = 0, t, \vec{r}) = v_i^t r_i r_{n+i} + (1 - v_i^t)(1 - r_i r_{n+i}).$$

To estimate the vector of parameters $\vec{r}$, the logarithm of likelihood function $F_T(\vec{r})$ is maximized, subject to the following constraints:

$$r_i \in [0, 1], \quad i = 1, 2, \ldots, 2n + 1. \qquad (4)$$

To improve the fit, we require that the marginal probabilities of affirmative votes equal their observed counterparts. This additionally imposes n constraints:

$$r_i r_{n+i} + (1 - r_i)r_{2n+1} = p_i, \quad i = 1, 2, \ldots, n, \qquad (5)$$

where the means $p_i = (1/T) \sum_{t=1}^{T} v_i^t$ are the frequencies of Yes votes. We use the following re-parametrization to simplify the optimization problem: $R_i = r_i$, $R_{n+i} = r_i r_{n+i}$ and $R_{2n+1} = r_{2n+1}$, with $R_i \geq R_{n+i}$ imposed in addition to (4). The estimates were obtained using the interior-point methods available in the MATLAB Optimization Toolbox.

The Bahadur (1961) parametrization of the joint probability distribution of n Bernoulli random variables suggests many moment-based constraints that may additionally be imposed. A natural addition to the first-moment constraints (5) would be the $n(n-1)/2$ constraints based on the mixed moments $\mathbf{E}V_iV_j$ and the frequency of all ballots in which the legislators i and j both voted Yes of the form

$$\mathbf{E}V_iV_j = r_i r_{n+i} r_j r_{n+j} + (1 - r_i)r_j r_{n+j} r_{2n+1} +$$
$$+ r_i r_{n+i}(1 - r_j)r_{2n+1} + (1 - r_i)(1 - r_j)r_{2n+1}, \quad i, j = 1, 2, \ldots, n, \ i < j,$$

where the sample counterpart of the right side equals $(1/T) \sum_{t=1}^{T} v_i^t v_j^t$. Introducing additional moment-based constraints can substantially increase the complexity of the optimization problem and may lead, the constraints being contradictory, to the nonexistence of a solution. We found that imposing these additional constraints did not tangibly improve the accuracy of our model, although other applications may profit therefrom.

The above likelihood function implicitly assumes a fixed number of legislators deciding on every ballot, and attaches the index i to the same legislator. While the assumption of a constant composition may work for small voting bodies such as juries, it is not tenable for large voting assemblies such as parliaments. The actual number of votes cast on any ballot is likely to be smaller than the number of seats in the parliament, because some

legislators could abstain from voting, be temporarily absent or be permanently replaced by other legislators in the middle of a legislative session, whether due to resignation or demise.

Our method is sufficiently flexible to accommodate abstentions and irregular tenures. To account for absenteeism and abstentions in parliament, we introduce a binary participation parameter a_i^t, such that $a_i^t = 1$ if legislator i voted on the ballot t, and $a_i^t = 0$ if she did not. If $a_i^t = 0$, we set $v_i^t = 1$. This information is collected in an $n \times T$ binary attendance matrix A. The following definitions replace their counterparts in problem (3):

$$F(i, 1, t, A, \vec{r}) = a_i^t [v_i^t (1 - r_i(1 - r_{n+i})) + (1 - v_i^t) r_i(1 - r_{n+i})] + 1 - a_i^t, \tag{6}$$

$$F(i, 0, t, A, \vec{r}) = a_i^t [v_i^t r_i r_{n+i} + (1 - v_i^t)(1 - r_i r_{n+i})] + 1 - a_i^t, \tag{7}$$

where the means are now computed as

$$p_i = \frac{1}{\sum_{t=1}^{T} a_i^t} \sum_{t=1}^{T} a_i^t v_i^t.$$

This simple modification fully captures absenteeism and abstentions, as well as different tenures of legislators. If i has resigned during a session at time τ, then $a_i^\tau = 0$ for all $\tau \geq t$. If j succeeds i, then $a_j^\tau = 0$ for all $\tau < t$. In this formulation, n denotes the number of legislators that voted at least once. The estimates for the Swiss parliament below were obtained using an adjusted maximum likelihood function (3) with (6) and (7), under the moment restrictions (4) and (5).

3   Estimates and application

Swiss Parliament comprises two houses: a Lower House (National Council) and an Upper House (Council of States). This study analyzes voting behavior in the Lower House, which has 200 members elected using a proportional system. The members of the Lower House are elected by geographical constituencies – the 26 cantons of Switzerland. The Swiss constitution stipulates that members of the Lower House represent the entire people of Switzerland, as the name 'National Council' suggests. The constitutional mandate determines our choice of the common factor as the will of the national median voter. The members of the Upper House are explicitly referred to as the representatives of the cantons, hence the 'Council of States'.

Swiss legislators vote on new laws and amendments, as do representatives in other parliamentary democracies around the world. But Swiss direct democracy allows us to directly measure the congruence between representatives and the national median voter. The proposals accepted by the parliament do not necessarily become law. Parliamentary decisions can be challenged by citizens demanding a referendum. Final votes in the Lower House are recorded using an electronic voting system. We apply the estimation approach detailed in the previous section to final roll-call data on bills that were subsequently submitted to a country-wide referendum. In particular, these bills include all constitutional

amendments, as they require a confirmatory referendum. A majority of voters and cantons (Stände) suffices to change the constitution (Stephan and Cofone 2017). In addition to these constitutional referendums, a small group of citizens can put forward an initiative to amend the constitution by referendum, or demand a referendum on a simple law already passed by the parliament. In all cases the legislators vote on precisely the same proposals as the citizens, allowing us to compare their votes (Portmann 2014, Stadelmann, Portmann and Eichenberger 2018).

3.1 The probability of acting as a trustee

We estimate the model using roll-call data from the Swiss Lower House for three legislative sessions from 1999 to 2011. The estimated individual probability of acting as a trustee is given by $\mathbf{E}L_i = r_i$, the expected value of the mixing variable. Table 1 summarizes the distributions of the estimates by legislative session.

Depending on the legislative session, the median values (column q50) of the estimates of r_i lie between 0.48 and 0.62, implying that half of the legislators act as trustees slightly more often than half of the time. Figure 1 plots the estimated probabilities of acting as a trustee by session, sorted in ascending order. About one third of the estimates are equal to one, implying clear idiosyncratic behavior typical of trustees. We call the representatives whose estimated probability of acting as a trustee equals unity the *perfect trustees*. Who are the perfect trustees and how representative of the electorate are they? We shall return to this question after having explored the validity of the empirical model.

Table 1: Estimated Probability of Acting as a Trustee

Session	MPs	Refs	Min	q25	q50	q75	Max
1999 to 2003	212	43	0	0.13	0.54	1	1
2003 to 2007	224	20	0	0.32	0.48	1	1
2007 to 2011	220	30	0	0.21	0.62	1	1

The abbreviations Min, q25, q50, q75, Max denote the minimum, the 25 percent quantile, the 50 percent quantile (median), the 75 percent quantile and the maximum, respectively.

3.2 The probability of congruence – model validation

To validate the model, we compare the estimated probability of congruence with the observed frequency of congruence, which is obtained by matching the representative's roll-call vote with the decision of the national median voter in each referendum.

Table 2 summarizes the distributions of the *estimated* individual probabilities of congruence (columns titled ESTIMATES), i.e. $\mathbf{E}(V_i = M)$. The estimated total probabilities range from 0.37 to 1. Median values between 0.69 and 0.76 suggest that half of the legislators are estimated to disagree with the median voter in about 30 percent of their decisions. This figure is consistent with the observed congruence rates when comparing the representatives and voters in referendums (Brunner et al. 2013, Garrett 1999, Stadel-

mann et al. 2013, Matsusaka 2015). The extreme estimates of one, indicating perfect congruence, occur for legislators with exceptionally short tenures. In our dataset, this applies to four members of the 1999-2003 session and five members of the 2007-2011 session, who voted on fewer than one-tenth of ballots during a session – too seldom for a reliable estimate.

The model is estimated using the entire voting record of a given session on proposals with subsequent referendums, yet the above probabilities can be obtained separately for Yes and No votes. The median probability of congruence for the first and third session is higher for No votes than for Yes votes, suggesting that legislators are more attentive to voters if they are likely to disapprove. The estimates indicate that 46, 24 and 31 legislators in the respective sessions flawlessly anticipated the disapproval of the majority, resulting in absolute congruence. The corresponding numbers for the Yes votes are 17, 36 and 17.

We *validate* the estimation model using correlation coefficients between the estimated probability and the observed frequency of congruence. We also consider the coefficient of determination in a logistic regression of the observed frequency of congruence on the estimated probability. The observed frequency of congruence reflects the actual matches between the vote of a representative and the observed will of the national median voter. These are the cases in which the representative voted Yes and the subsequent referendum resulted in a Yes, or in which the legislator voted No and the referendum resulted in a No. It is important to emphasize that this information has not been used in the estimation of the probability of congruence, as it would normally not be available due to the absence of referendums as an instrument of direct democracy.

Table 2: Estimated Probability of Congruence

Session	MPs	Refs	DATA Votes	ESTIMATES Min	q25	q50	q75	Max	VALIDATION ρ	τ	R^2
1999 to	212	43	All : 7458	0.48	0.49	0.74	0.93	1	0.89	0.61	0.72
2003			Yes : 3941	0.68	0.82	0.87	0.96	1	0.83	0.68	0.56
			No : 3517	0.48	0.67	0.94	0.99	1	0.96	0.59	0.86
2003 to	224	20	All : 3646	0.37	0.47	0.76	0.83	0.99964	0.69	0.51	0.48
2007			Yes : 2214	0.78	0.86	0.89	0.97	1	0.76	0.64	0.42
			No : 1432	0.37	0.62	0.81	0.91	1	0.88	0.7	0.72
2007 to	220	30	All : 5391	0.49	0.5	0.69	0.89	1	0.63	0.56	0.36
2011			Yes : 2830	0.69	0.8	0.84	0.95	1	0.49	0.45	0.18
			No : 2561	0.58	0.71	0.87	0.94	1	0.78	0.54	0.49

We validate the model by comparing the estimated probabilities of congruence by correlation coefficients (Pearson ρ and Kendall τ) with the observed frequencies of congruence. We also employ a pseudo-coefficient of determination for a logistic regression of the frequencies of congruence on the probabilities (Nagelkerke's $R^2 \in [0,1]$) for validation.

In Table 2 (columns titled VALIDATION), ρ denotes the standard Pearson product-moment coefficient and τ denotes the Kendall rank correlation. The input of both correlation coefficients is the estimated probability and the observed frequency of congruence. The Kendall rank correlation coefficient τ is better suited for uncovering dependence in a

nonlinear relationship. Both correlation coefficients indicate a strong association between the estimated probability and the observed frequency of congruence. The correlation patterns are broadly consistent, except for the relative strength of the correlation with the No votes during 1999-2003. Nagelkerke's R^2 serves as a measure of fit for a (cross-sectional) logistic regression of the estimated probability of congruence on the observed frequency of congruence; it confirms good cross sectional fits implied by the correlation analysis. The fits are better for No votes than for Yes votes. The model was thus able to predict reasonably accurately the actual matches between the decisions of individual legislators and the will of the majority of voters. The validation exercise suggests that our empirical model can be used to infer congruence levels of politicians, even if the will of the national median voter is unobserved. Plausible cross-sectional estimates for the probabilities of acting as a trustee or a delegate further lend credibility to the model.

3.3 Ideology and acting as a trustee – an application

The possibility of estimating individual inclinations to act as trustees or delegates opens several potential applications. By way of an example, we consider the statistical relationship between the probability of acting as a trustee and the ideological positions of the representatives.

Table 3: Estimated Probability of Acting as a Trustee by Ideology Blocks

| Session | MPs | Block | ESTIMATES | | | | | SCORES | | |
			Min	q25	q50	q75	Max	q50	ρ	R^2
	66	Left	0.89	1	1	1	1	-8.18		
1999 to 2003	88	Center	0	0.06	0.13	0.21	1	1.53	-0.62 (0.26)	0.42 (0.07)
	42	Right	0.19	0.49	0.56	0.63	1	7.97		
	75	Left	0.82	1	1	1	1	-8.36		
2003 to 2007	82	Center	0	0.14	0.29	0.47	1	1.80	-0.74 (-0.2)	0.56 (0.04)
	52	Right	0.23	0.40	0.45	0.48	0.99	8.02		
	67	Left	0.83	1	1	1	1	-8.48		
2007 to 2011	76	Center	0	0.07	0.14	0.28	0.94	1.64	-0.52 (0.17)	0.29 (0.3)
	62	Right	0.17	0.53	0.62	0.68	1	8.12		

The NOMINATE ideology scores range from -10 (left) to +10 (right). The total number of representatives by session is lower than in Table 1, because we do not have the scores for 46 MPs. ρ is the Pearson correlation coefficient between the ideology scores and the probability of acting as a trustee, and $R^2 \in [0, 1]$ is the Nagelkerke's pseudo-coefficient of determination for a logistic regression of the probabilities on the scores for all observations belonging to a legislative session. The figures in parenthesis were computed using the sample that excludes the perfect trustees.

Table 1 shows that over 25 percent of the estimates equal unity (column q75), implying clear idiosyncratic behavior. Who are the perfect trustees? To answer this question, we explore the relationship between the ideology and the probability of acting as a trustee by using the NOMINATE ideology scores of the representatives.[3] The scores range from

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[3]The NOMINATE scores by Michael Hermann of the Sotomo Institute are featured in the newspaper

-10 (left) to +10 (right). Like the probabilities of acting as a trustee, the scores were obtained using the roll-call data only. Since the score of a representative can change over time, we use its value at the time of voting in parliament.

Figure 2 plots the estimated probabilities of acting as a trustee against NOMINATE ideology scores, separately for each legislative session. Three clusters appear clearly in each of the three legislative sessions. We observe a leftist block of perfect trustees (crosses), a centrist block whose members mostly act as delegates (empty bullets), and a rightist block (filled bullets) whose members act as delegates about half of the time. The left block mainly comprises Greens (GPS) and Social Democrats (SP). The centrist block is the most diverse of the three. It includes the Green Liberals (GLP), Christian Democrats (CVP), Conservative Democrats (BDR) and Liberals (FDP). The right block mainly includes the members of the Swiss People's Party (SVP).

Table 3 provides a closer look at the members comprising the three blocks. Although perfect trustees exist in each block, they are concentrated in the political left of the ideology spectrum, whereas representatives from centrist parties tend to act as delegates. This is not entirely surprising. Having sufficiently many parliamentary parties that are ideologically distinct increases the likelihood of locating the perfect trustees on the opposite half of the spectrum relative to the median voter.

Since the representatives are elected by cantons, they have an incentive to put the preferences of their canton before those of the nation, despite a clear mandate to represent the entire nation. Could the perfect trustees in fact have been acting as delegates of their cantons? The average frequencies of congruence in Table 4 show that our data do not support this conclusion. The average congruence rates of perfect trustees at the cantonal level were quite low at 48.9 percent, while the cantonal and national referendum outcomes coincided in 88.5 percent of all parliamentary decisions involving the perfect trustees. This means that perfect trustees have similar congruence rates at the cantonal and nation levels, which, in turn, implies that they have not acted as delegates of their cantons while acting as trustees with respect to the nation.

Table 4: Average Frequencies of Congruence for Perfect Trustees

Session	MPs	MP vs. Canton	Canton vs. Nation
1999 to 2003	65	39.7	89.6
2003 to 2007	71	57.9	87.7
2007 to 2011	60	56.5	87.3

The averages in percent refer to the subset of parliamentary decisions involving at least one perfect trustee. The overall averages are, respectively, 48.9 and 88.5 percent.

The relationship between the ideological position and the probability of acting as a trustee can be further explored using the methods that were used to validate the model. A negative correlation coefficient between the ideology score and this probability means that moving to the right of the spectrum decreases the likelihood of trustee behavior. The

article 'Die Parteien sprechen mit einer Sprache', Neue Zürcher Zeitung of 25.11.2014.

Pearson product-moment coefficient ρ is negative and rather high in absolute value. The Nagelkerke's R^2 for the (cross-sectional) logistic regression of the probabilities of acting as a trustee on the NOMINATE scores shows that the scores can explain between 30 and 60 percent of the sample variation. Excluding perfect trustees from the sample suggests that in the Swiss roll-call data the overall relationship between the ideological position and the probability of acting as a trustee remains ambiguous. A more extensive investigation employing our estimation method could further look at what makes a representative a perfect trustee, such as personal characteristics or political experience. This is a promising venue for further research, since analysis of Swiss referendums by Bjedov, Lapointe and Madiès (2014) documents nontrivial differential effects of ideological dispersion within and across parties on fiscal policy outcomes.

4   Conclusions

We propose a new empirical approach for estimating the probability of a political representative acting as a trustee. The underlying assumption is that trustees vote idiosyncratically. Under this assumption, we can estimate the probability from a set of roll-call data only. The fact that the estimates are specific to each political representative opens the venue for investigations that not only address the nature of issues, but also political campaigns, party affiliations and the personal characteristics of representatives. In our empirical application, we find that most of the Swiss representatives who act as perfect trustee tend to come from the left side of the traditional ideology spectrum.

The second quantity of interest is the probability of a representative voting according to an unobserved common signal that influences the votes of all representatives. If the common signal reflects the will of the national median voter, this would be the probability of congruence with the median voter. We validate the model and the assumption of the common signal representing the will of the Swiss median voter by comparing the predicted probabilities of congruence with the observed frequencies of congruence. The frequencies were obtained by matching the roll-call votes of Swiss legislators with referendum outcomes. The predicted probabilities of congruence strongly correlate with observed frequencies of congruence. Based on this evidence, we conclude that the model can be used for estimating the country-wide congruence levels for individual political representatives based on the roll-call votes alone, as well as for inferring whether legislators are acting as delegates or trustees.

The model has several attractive features: i) it is flexible enough to accommodate abstentions and irregular tenures that are common in parliaments, ii) it generates a positive correlation between individual votes that is typically observed, iii) it delivers estimates at an individual level (ranking of politicians) that can be aggregated at the institutional level. This allows us to estimate the fidelity of representation for each legislator and for the parliament, under the assumption that the common signal represents the will of the national median voter.

The model discussed in this paper uses the simplest dichotomy of unobserved random

factors (signals): purely idiosyncratic individual random effects combined with a single random effect common to all individuals. This dichotomy is sufficient, given our focus on the probability of acting as a trustee. The remaining analysis validates the model and takes a first step towards empirically identifying the common factor. Upon establishing the usefulness of a single common factor for explaining voting patterns, the model can be extended to include additional unobserved factors that are common to a subset of the legislators. What is required is a discriminatory variable that groups legislators in a meaningful way and a sufficient number of observations in the voting data as a crucial ingredient necessary for sound statistical inference. We have pointed to the effect of a constituency (canton) as a prime candidate for an additional unobserved factor, as the median voter position of the home constituency is expected to exert significant influence on the voting behavior of the legislators. Including cantonal effects would allow predicting the probability of congruence with a median voter of the constituency, in addition to the probability of congruence with the national median voter. Further factors could, for example, be related to the unobserved determinants of party cooperation (opposition) or the influence by interest groups. To summarize, the model presented in this paper allows to successively refine the common signal from a perspective by making it, loosely speaking, successively less common. Since the estimation of rich binominal mixture models that contain multiple levels of signals requires extensive modeling and computing resources, our approach points to promising technical and empirical research avenues.

References

Ashworth, S.: 2012, Electoral accountability: Recent theoretical and empirical work, *Annual Review of Political Science* **15**, 183–201.

Bahadur, R. R.: 1961, A representation of the joint distribution of responses to n dichotomous items, *in* H. Solomon (ed.), *Studies in item analysis and prediction*, Stanford University Press, pp. 158–168.

Barceló, J.: 2019, The clarity of the majority's preference moderates the influence of lobbying on representation, *Party Politics* **25**(3), 349–357.

Bjedov, T., Lapointe, S. and Madiès, T.: 2014, The impact of within-party and between-party ideological dispersion on fiscal outcomes: Evidence from Swiss cantonal parliaments, *Public Choice* **161**(1/2), 209–232.

Brunner, E. J., Ross, S. L. and Washington, E. L.: 2013, Does less income mean less representation?, *American Economic Journal: Economic Policy* **5**(2), 53–76.

Burke, E.: 1774 (2013), *The works of the right honorable Edmund Burke*, HardPress Publishing.

Fox, J. and Shotts, K. W.: 2009, Delegates or trustees? A theory of political accountability, *The Journal of Politics* **71**(4), 1225–1237.

Garrett, T. A.: 1999, A test of shirking under legislative and citizen vote: The case of state lottery adoption, *Journal of Law and Economics* **42**(1), 189–208.

Gerber, E. R. and Lewis, J. B.: 2004, Beyond the median: Voter preferences, district heterogeneity, and political representation, *Journal of Political Economy* **112**(6), 1364–1383.

Golder, M. and Ferland, B.: 2018, Electoral systems and citizen-elite ideological congruence, *in* E. S. Herron, R. J. Pekkanen and M. S. Shugart (eds), *The Oxford Handbook of Electoral Systems*, New York: Oxford University Press.

Grofman, B., Owen, G. and Feld, S. L.: 1983, Thirteen theorems in search of the truth, *Theory and Decision* **15**(3), 261–278.

Hessami, Z.: 2016, How do voters react to complex choices in a direct democracy? Evidence from Switzerland, *Kyklos* **69**(2), 263–293.

Iaryczower, M. and Shum, M.: 2012, The value of information in the court: Get it right, keep it tight, *American Economic Review* **102**(1), 202–237.

Matsusaka, J. G.: 2010, Popular control of public policy: A quantitative approach, *Quarterly Journal of Political Science* **5**(2), 133–167.

Matsusaka, J. G.: 2015, The precarious link between legislators and constituent opinion: Evidence from matched roll call and referendum votes. University of Southern California.

Matsusaka, J. G.: 2018, Public policy and the initiative and referendum: A survey with some new evidence, *Public Choice* **174**(1), 107–143.

Pitkin, H. F.: 1967, *The concept of representation*, Los Angeles: University of California Press.

Poole, K. T. and Rosenthal, H.: 1997, *Congress - A political-economic history of roll call voting*, New York: Oxford University Press.

Portmann, M.: 2014, *Parliamentary representation of citizens' preferences: Explaining the differences between parliamentarians' votes and popular referendum results*, Vol. 15 of *Neue Studien zur politischen Ökonomie*, 1 edn, Nomos, Baden-Baden.

Przeworski, A., Stokes, S. C. and Manin, B. (eds): 1999, *Democracy, accountability, and representation*, Cambridge University Press.

Stadelmann, D., Portmann, M. and Eichenberger, R.: 2013, Quantifying parliamentary representation of constituents' preferences with quasi-experimental data, *Journal of Comparative Economics* **41**(1), 170–180.

Stadelmann, D., Portmann, M. and Eichenberger, R.: 2018, Preference representation and the influence of political parties in majoritarian vs. proportional systems: An empirical test, *British Journal of Political Science* **49**(1), 181–204.

Stephan, M. and Cofone, I. N.: 2017, Majority rules in constitutional referendums, *Kyklos* **70**(3), 402–424.

Stokes, S. C.: 2001, *Mandates and democracy: Neoliberalism by surprise in Latin America*, Cambridge University Press.

Figure 1: Estimated Probability of Acting as Trustee
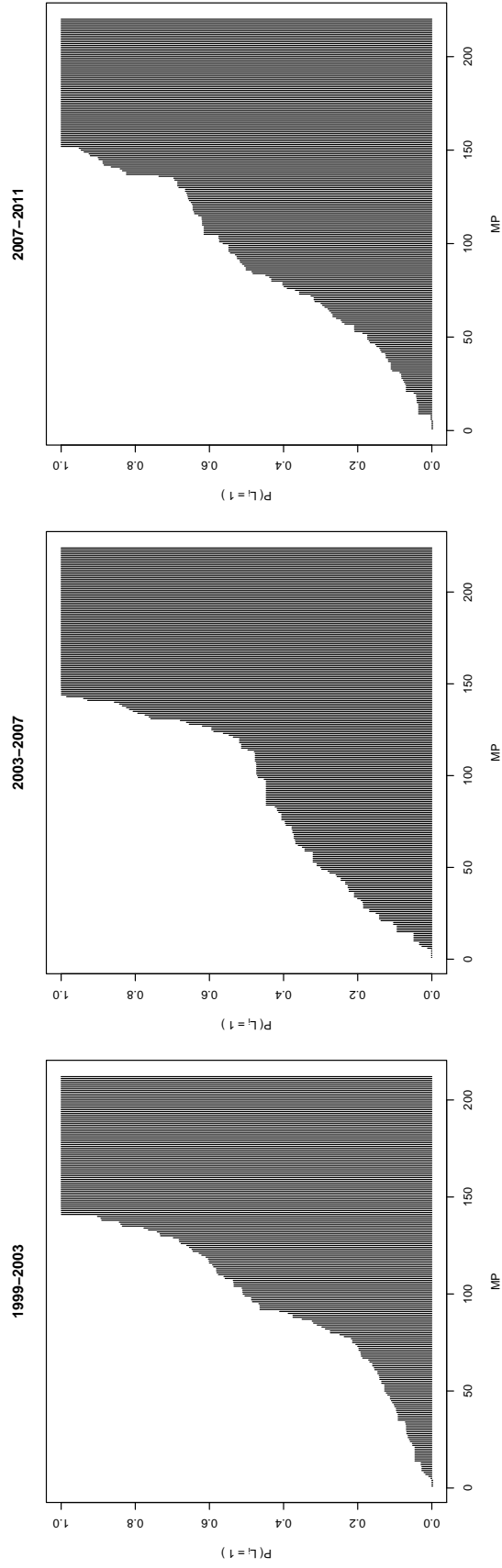
The probability of a legislator acting as a trustee is given by $\mathbf{E}L_i = r_i$, the expected value of the mixing variable. The estimates are sorted in ascending order of their magnitude. Roughly one third of the estimates equals unity, implying a certain idiosyncratic behavior characteristic of trustees.

Figure 2: Ideology Scores vs. Probability of Acting as Trustee
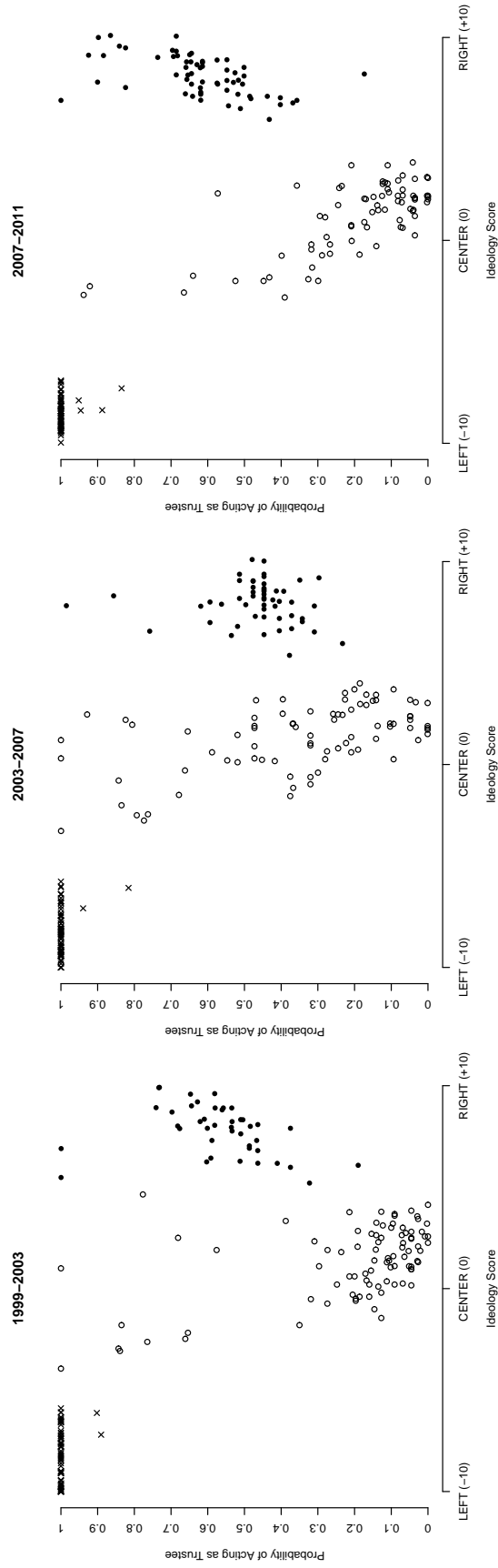
A plot of estimated probabilities of acting as a trustee against NOMINATE ideology scores reveals three clusters: a leftist block of perfect trustees (×), a centrist block whose members mostly act as delegates (○), and a rightist block (●) whose members act as delegates about half of the time.

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